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Borrowing with Unobserved Liquidity Constraints: Structural Estimation with an Application to Sovereign Debt

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ABSTRACT

Borrowing with Unobserved Liquidity Constraints: Structural Estimation with an Application to Sovereign Debt

We develop a framework for estimating the optimal expenditure of agents subject to unobserved liquidity constraints. Our framework allows us to estimate credit ceilings as well as preference parameters. We apply the framework to data on net resource transfers from private lenders to twenty-nine sovereign debtors during 1973-1993. We obtain reasonable estimates of the discount factor, elasticity of marginal utility of expenditure, and the credit ceiling for most countries. Our estimated credit ceilings rise quite regularly with income across the countries of our sample, and are positively associated with a country's trade, in line with several theoretical arguments. Our estimates imply that slightly less than half the countries in our sample were liquidity constrained during the 1970s. The fraction rose to around 80 per cent in the mid 1980s, and subsequently declined.

RESUME

Contraintes de liquidité inobservées: estimation structurelle et application à la dette des PVD.

Cette article étudie la consommation et l'emprunt optimal lorsqu'un agent fait face à une contrainte de liquidité et un taux d'intérêt stochastique. Nous développons une méthode d'estimation qui permet d'identifier les plafonds d'endettement non observés par l'économètre, ainsi que les paramètres de préférences des agents. Nous appliquons la méthode à un panel de 29 pays en voie de développement, entre 1973 et 1993. Nous obtenons des estimations raisonnables pour l'élasticité de l'utilité marginale de la consommation pour la plupart des pays, ainsi que pour les plafonds d'endettement. Ceux ci sont positivement corrélés avec le revenu ainsi qu'avec l'ouverture économique du pays, corroborant des travaux théoriques. Nos estimations montrent qu'un peu moins de la moitié des pays étudiés étaient contraints dans les années 1970. Cette proportion atteint 80% dans la décénie suivante, puis a décru dans les années 1990.

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1 Introduction

Liquidity constraints often appear to limit borrowing. In particular, strong evidence indicates that two very different types of parties, sovereign governments of developing countries and households in developed countries face such constraints. While we observe substantial amounts of borrowing both by sovereign countries and by households, both types of borrowers appear to face limits on how much they can go into debt.

In the case of sovereign debt, the remedies that creditors have in the event of nonpayment are unlikely to enforce the repayment of debt anywhere near the level needed to smooth expenditure optimally. Hence there are strong theoretical reasons to suspect that lenders impose constraints on how much sovereign governments borrow in the first place. During various historical periods the governments of developing countries have borrowed substantial sums of money, and repaid much of what they borrowed. The net resource transfers in either direction have occasionally exceeded 5 per cent of the borrower's GDP, but they have been far too small to smooth expenditure optimally: Expenditure has remained very closely tied to income.[1] While much work undertaken during the last decade and a half has attempted to provide a theory of sovereign lending in the presence of these constraints, it has not yet been used to provide a structural specification for empirical estimation of borrowing and repayment. [2] There are at least two reasons for this gap between theory and estimation. One is that, until recently, time series on sovereign debt have been too short to allow estimation of the relevant parameters. By now, however, enough time has passed since data on sovereign indebtedness were first collected systematically to allow an examination of the time-series properties of this lending. A second reason is that the theoretical models of sovereign indebtedness are dynamic and highly nonlinear, and fail to provide closed-form relationships between debt and other observables that can be estimated directly. While the cross-section analysis in Eaton and Gersovitz (1981) suggests that credit rationing during 1971 and 1974 was pervasive, their specification can at best be interpreted as identifying broad reduced-form relationships suggested by the theory.

The household consumption literature has taken the empirical investigation of credit constraints further. Several studies look for liquidity constraints by testing for the excess sensitivity of consumption to current income, typically using a quadratic framework.[3] One study which does relax the quadratic utility assumption is Zeldes

[1] As indicated in Table 1, transfers have reduced the variance of expenditures around trend by only insignificant amounts. Cohen (1992) provides data from the last two decades on how much was borrowed, and on how much of what was borrowed was repaid. He reports that for some countries total debt peaked at more than 100 per cent of GDP.

[2] Eaton and Fernandez (1995) provide a recent survey of this literature. Eaton and Gersovitz (1981) developed a theory of potentially constrained borrowing for consumption-smoothing purposes which they used as a basis for estimating the determinants of indebtedness of a cross section of sovereign debtors. Kletzer and Wright (1995) provide a much more general model of sovereign borrowing with constrained credit.

[3] Significant contributions to this literature are Hall and Mishkin (1982), using household data, and Flavin (1985) and Campbell and Mankiw (1989), using aggregate data. An analytic solution requires a quadratic specification, but this functional form rules out any precautionary motive for saving. This omission can lead to the false acceptance of liquidity constraints. See Carroll (1992)

(1989a). In order to identify the model, however, he has to make an *ad hoc* assumption about which agents are subject to binding constraints. The inability to infer which agents are constrained from the data is a serious shortcoming of the standard Euler equation approach. Moreover, all these papers only *test* for the presence of liquidity constraints and do not provide any insight on the levels of the credit ceilings or their determinants. Taking a different approach, Deaton (1991) simulates the optimal consumption of an agent with isoelastic utility facing a borrowing constraint. He shows that imposing such a constraint makes it much easier to reconcile observed consumption behavior with consumer optimization. He does not estimate the parameters of his model, however. Moreover, his simulations simply impose a credit ceiling of zero, i.e., his consumer is never allowed to be a net debtor.

In summary, evidence from both sovereign debt and from household consumption suggests that borrowers face limits on how far into debt they can fall. We cannot observe these limits directly, however, and so far have inferred their magnitudes only by speculation.

In this paper we provide a methodology for estimating spending by a credit constrained agent which allows us to infer the size of the constraint itself. We apply the methodology to borrowing by a sample of twenty-nine indebted developing countries during the period 1973-1993. An essential feature of these countries' participation in international capital markets was their ability to incur *net* debt, forcing us to do away with the assumption that their net external wealth could not be negative. Instead, we introduce, as an unobserved (by us) state variable, a stochastic credit ceiling for each country. We estimate the parameters characterizing the distribution of this state variable, along with the parameters of the utility function. Since interest rate movements were quite large during this period, we also introduce a time-varying real interest rate as an additional state variable. The borrower has access to loans at an exogenous safe, but time-varying, interest rate. It can borrow and lend as much as it wants at this rate subject to a standard transversality condition and a less standard requirement that its debt not exceed a particular limit. Its objective is to maximize the present discounted utility of current and future expenditure. [4]

The estimation method used in this paper follows the approach of Deaton and Laroque (1996). Instead of relying on the first-order condition, we base our estimations on the fully-solved model. To this end, we characterize the stochastic, rational-expectation equilibrium which relates the optimal level of expenditure to the state variables, which here are income (defined here as GDP plus official transfers), the real interest rate, and credit ceiling. We assume that behavior maximizes an isoelastic (constant relative risk aversion) utility function. Since it is non-quadratic, there is no closed-form solution for optimal expenditure. Hence we rely on numerical methods. For each of twenty-nine countries we estimate four parameters: the elasticity of the marginal utility of expenditure, the discount factor, and the mean and variance of the credit ceiling. Parameter estimates minimize the squared deviation between actual and predicted expenditure given the observed path of debt.

for a discussion.

[4]In fact, the interest rates charged to these countries typically incorporated an explicit risk premium. To the extent that this premium reflects the possibility of nonpayment, it does not constitute part of the marginal cost of borrowing. See Eaton and Gersovitz (1987) for a discussion of the relationship between the interest rate and the marginal cost of capital.

Adding credit constraints to the simple Ramsey model allows us to explain the borrowing behavior of most of the countries in our sample quite well. The model usually yields plausible estimates of the parameters of the utility function and explains most of the variability in the data. Our estimates of the credit ceiling run from about 5 to around 50 per cent of mean income over the sample. Regressing our estimates of these credit ceilings on country characteristics associated with creditworthiness we find that a percentage increase in a country's mean income raises its credit ceiling by about 1.6 per cent. Moreover, the credit ceiling is positively related to a country's degree of openness, as measured by the sum of its exports and imports as a share of GDP. The credit ceiling is also negatively related to the variance of the income innovation.

We calculate the probability with which each country was up against the credit constraint in each period of the sample. We find that about half of our countries were credit-constrained in the 1970s. This fraction began to rise in the early 1980s, peaked at about 80 per cent in 1985, and then declined back toward 50 per cent.

Finally, we calculate the effect of a (surprise) 10 per cent increase in the credit ceiling for each of our countries. The effect is typically to raise welfare by about .1 per cent. Section 2 below describes our model of borrowing subject to a stochastic credit ceiling and discusses how to solve its stochastic rational expectation equilibrium numerically. Section 3 then presents a procedure to estimate the parameters of the model. Section 4 describes how we apply this methodology to sovereign borrowing over the last three decades. Finally, section 5 concludes.

2 The Model

Here we present our basic assumptions. We then show that they give rise to a unique expenditure function, and describe some of its features.

We assume that the behavior of a borrower at time t can be described in terms of its efforts to maximize an intertemporal utility function:

$$W_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(e_\tau)$$

where e_τ denotes expenditure in period τ.[5] We assume that $u(e_\tau)$ has the standard isoelastic form:

$$u(e_\tau) = \frac{e_\tau^{1-\gamma}}{1-\gamma}$$

where γ is elasticity of the marginal utility of income, which measures the concavity of the utility function. It reflects the risk aversion, or "prudence," of the borrower, and β is a discount factor. Each period τ the borrower has an (exogenous) realization of income y_τ. At the beginning of period τ it has (potentially negative) net claims equal to $R_{\tau-1}a_{\tau-1}$, where $a_{\tau-1}$ is the borrower's net claims at the end of period $\tau-1$ and $R_{\tau-1} = 1 + r_{\tau-1}$, where $r_{\tau-1}$ is the real interest rate . We depart from the standard intertemporal optimization framework in assuming that creditors impose

[5] Expenditures include investment and government spending as well as consumption.

a credit ceiling that prevents a_τ from falling below a (presumably nonpositive) floor $A(z_\tau)$, Here z_τ denotes all (exogenous) variables on which the credit ceiling could depend.[6] Hence net claims evolve according to:

$$a_\tau = R_{\tau-1}a_{\tau-1} + y_\tau - e_\tau \qquad a_\tau \geq A(z_t). \tag{1}$$

(To simplify the notation, we drop, where obvious, the conditioning variables z_t as arguments of A_t.) We explore the set of variables z_t which might influence the lending in section 4.

If the liquidity constraint does not bind, then the standard Euler condition for a maximum implies that:

$$e_\tau^{-\gamma} = \beta R_\tau E_\tau \left[e_{\tau+1}^{-\gamma}\right] \tag{2}$$

where E_τ is the expectations operator as invoked in period τ. It may be the case, however, that to satisfy (2) the borrower must violate its liquidity constraint. The maximum that the borrower can spend and not violate its liquidity constraint is:

$$x_\tau = R_{\tau-1}a_{\tau-1} + y_\tau - A_\tau, \tag{3}$$

which, following the literature, we refer to as "cash-on-hand," although, with our generalization, this amount now includes whatever cash can be raised by borrowing. Note that the expression for cash-on-hand is just as in Deaton (1991) except that we have added that period's credit ceiling ($-A_\tau$) to wealth and income. Cash-on-hand evolves according to

$$x_\tau = R_{\tau-1}(x_{\tau-1} - e_{\tau-1}) + y_\tau - A_\tau + R_{\tau-1}A_{\tau-1}. \tag{4}$$

Given that expenditure cannot exceed cash-on-hand, the first-order condition for a maximum becomes:

$$e_\tau^{-\gamma} = \max[x_\tau^{-\gamma}, \beta R_\tau E_\tau e_{\tau+1}^{-\gamma}]. \tag{5}$$

If the borrower is constrained then expenditure equals cash-on-hand, and the borrower is at its debt ceiling at the end of the period. Otherwise, borrowing satisfies the standard Euler condition, except that the expectations operator takes into account the possibility that the constraint might bind in the future. The optimization is consequently significantly more complicated, as the borrower must take current and future borrowing constraints into account in deciding how much to spend. The borrower's situation at any time t can be described in terms of cash-on-hand x_t, and the set of exogenous state variables $s_t = \{y_t, R_t,\ A_t\}$, where y_t is income, R_t the interest rate, and A_t the credit ceiling.

2.1 Introducing Trends

To accommodate the fact that income, wealth, and debt tend to grow over time, we introduce a common deterministic trend to income and the credit ceiling:

[6]Note that we allow the credit ceiling to be time varying and stochastic, whereas Deaton (1991) imposes it to be constant at zero in his simulations.

Assumption 1

a. *Income has a deterministic exponential trend: $y_t = e^{\alpha t}\widetilde{y}_t$, where $\widetilde{y}_t$ is a stationary random variable with properties we impose below. The credit ceiling has the same trend as income: $A_t = e^{\alpha t}\tilde{A}_t(z)$, where $\tilde{A}_t(z)$ is a random variable with mean $\mu_A(z)$ and variance $\sigma_A^2(z)$.*

The level of the credit ceiling and its variance depend on a vector z of characteristics of the borrower, which we consider as fixed over the sample period (variance of GDP, degree of openness, reliance on oil exports, etc.). Although the estimation could in principle accommodate a richer stochastic specification for the credit ceiling, we make this simplifying assumption to reduce the number of parameters to estimate, since the sample period is probably not long enough to identify the transition probabilities associated with A_t. Therefore, we restrict the deterministic component of A_t to follow the same path as income, which seems to us intuitive, as long-run income is probably the main determinant of lending.

We denote the detrended variables with a tilde, i.e., for any variable v, $\widetilde{v}_t = v_t e^{-\alpha t}$. Incorporating the exponential trend into equations (1), (3), (4), and (5) the first-order condition becomes:

$$\tilde{e}_t^{-\gamma} = \max[\tilde{x}_t^{-\gamma}, \widetilde{\beta} R_t E_t \tilde{e}_{t+1}^{-\gamma}], \tag{6}$$

where $\widetilde{\beta} = \beta \exp(-\gamma\alpha)$, and cash-on-hand evolves as:

$$\begin{cases} \tilde{x}_t = \widetilde{R}_{t-1}\tilde{a}_{t-1} + \tilde{y}_t - \tilde{A}_t \\ \tilde{x}_t = \widetilde{R}_{t-1}(\tilde{x}_{t-1} - \tilde{e}_{t-1} + \tilde{A}_{t-1}) + \tilde{y}_t - \tilde{A}_t \end{cases} \tag{7}$$

where $\widetilde{R}_{t-1} = R_{t-1}e^{-\alpha}$. Thus, apart scaling $t-1$ dated variables by the factor $e^{-\alpha}$, the equations of the detrended model are much as in the basic model.

2.2 The Equilibrium

Our estimation procedure requires computing the expenditure function. However, as it has no closed-form solution, we must approximate it numerically. To ensure its existence and to characterize some of its basic properties, we make further assumptions about the processes generating the exogenous variables:

Assumption 2

a. *Detrended income $\tilde{y}_t$ is an autoregressive process:*

$$\tilde{y}_t = \rho_Y \tilde{y}_{t-1} + (1-\rho_Y)\mu_Y + \epsilon_t, \qquad V(\epsilon_t) = \sigma_Y^2 \tag{8}$$

where ϵ_t is an i.i.d. variable with compact support with lower band $\underline{\epsilon}$ and upper bound $\bar{\epsilon}$.

b. *The interest rate is an autoregressive process:*

$$R_t = \rho_r R_{t-1} + (1-\rho_r)\mu_r + \eta_t, \qquad V(\eta_t) = \sigma_r^2 \tag{9}$$

where η_t is an i.i.d. variable with compact support with lower band $\underline{\eta}$ and upper bound $\bar{\eta}$.

In our application to sovereign borrowing, we test these assumptions. We find that 2 (a) holds at the 5% confidence level for almost all countries in the sample. Assumption 2 (b) also holds at the 5% level. For cash-in-advance to remain stationary relative to trend income, the borrower cannot want to accumulate assets indefinitely. The following restriction on the discount factor and interest rate process ensures the existence of a stationary solution while allowing for occasional realizations of the interest rate in excess of the discount rate:

Assumption 3 *There exists a finite N such that for any s_t*

$$\widetilde{\beta}^N E[\prod_{i=1}^{N} R_{t+i}|s_t] < 1.$$

Under these assumptions we can establish that the following result, generalizing those in Deaton and Laroque (1992) or Chambers and Bailey (1996) by allowing for a stochastic interest rate and a stochastic credit ceiling:

Proposition 4 *Under assumptions 1, 2, and 3 there exists a unique stationary, rational expectations solution for expenditure given by the function $\widetilde{e}_t = e(\widetilde{x}_t, \widetilde{y}_t, R_t, \widetilde{A}_t)$. Expenditure increases monotonically in $\widetilde{x}$, $\widetilde{y}$, and $\widetilde{A}$. There exists a cut off limit $x^*(\widetilde{y}, \widetilde{R}, \widetilde{A})$ such that $e(\widetilde{x}, \widetilde{y}, R, \widetilde{A}) = \widetilde{x}$ for $\widetilde{x} < x^*(\widetilde{y}, R, \widetilde{A})$ and $e(\widetilde{x}, \widetilde{y}, R, \widetilde{A}) \leq \widetilde{x}$ for $\widetilde{x} \geq x^*(\widetilde{y}, R, \widetilde{A})$. The cutoff limit x^* is decreasing in $\widetilde{y}$ and $\widetilde{A}$ and increasing in R.*

Proof: See appendix A.

That is, cash-on-hand $\widetilde{x}$ has a cut-off limit x^* which depends on income, the interest rate, and the credit ceiling. For realizations of $\widetilde{x}$ below x^* the agent simply spends $\widetilde{x}$, while for $\widetilde{x}$ above x^*, expenditure is governed by condition (2). Figure 1 depicts an expenditure function calculated for given values of $R, \widetilde{y}$, and $\widetilde{A}$. The function is equal to the 45 degree line for cash-on-hand lower than the cut off limit.

3 Estimation Procedure

Another way of expressing equation (5) is to write it as a function of the Lagrange multiplier associated with the liquidity constraint, μ_t.

$$\widetilde{e}_t^{-\gamma} = \widetilde{\beta} R_t E_t \widetilde{e}_{t+1}^{-\gamma} + \mu_t \qquad (10)$$

If the Lagrange multiplier or, equivalently, the credit ceiling $\widetilde{A}_t$, were observed then equation (10) or (5) can be estimated by a generalized method of moments as in Hansen and Singleton (1982). The problem is that the Lagrange multipliers are very unlikely ever to be observed. Hence the first-order condition is useless as is.

One way out is to find periods or agents for which μ_t is known to be zero and to estimate the model on the selected subsample. Additional information is therefore needed. This is the approach taken by Zeldes (1989a) on the basis of observed wealth. A problem, however, is that wealth is endogenous, creating a sample selection problem.[7] Moreover, this method cannot identify the credit ceiling $\widetilde{A}_t$ itself, which is of substantial interest.

Another, less ambitious approach looks at reduced forms which identify some combination of the parameters. This is the approach taken by Deaton and Laroque (1992) or Chambers and Bailey (1996). The problem, of course, is that, again, individual parameters of interest are not identified. The approach taken here is different, building along the lines of Deaton and Laroque (1996), although we allow a much richer state space. We solve the model to derive the Marshallian demand functions for expenditure, which are functions of the observed state variables. The parameters characterizing the distribution of the unobserved credit ceiling $\widetilde{A}_t$ are then identifiable and can be estimated along with the other parameters. The gain comes at a cost, in both the numerical computation of the Marshallian demand function for expenditure and in imposing specific forms for the distribution of the exogenous shocks. This computation requires numerical calculations as there is no closed-form solution.

3.1 Parameter Estimation

We estimate the parameters of the model using a time series of data on the interest rate R and, for each borrower, time series on income y and net assets a. The first stage of the procedure is: (i) to infer from the R series estimates of μ_R, ρ_R, and σ_R^2 and (ii) to infer from each y series estimates of $\mu_Y, \rho_Y, \sigma_Y^2$, and α and the detrended income series $\widetilde{y}$ for that borrower. The estimate of α is then used to obtain a detrended asset series $\widetilde{a}$ for that borrower. The second stage is to estimate, from the detrended income and asset series $\widetilde{y}$ and $\widetilde{a}$ and the parameters $\theta^1 = [\mu_R,\ \rho_R,\ \sigma_R^2, \mu_Y, \rho_Y, \sigma_Y^2, \alpha]$ obtained from the first stage, the vector of parameters $\theta^2 = [\gamma, \widetilde{\beta}, \mu_A, \sigma_A]$, where γ is the curvature of the utility function, $\widetilde{\beta}$ is the growth-adjusted preference rate, μ_A the mean of the credit ceiling and σ_A^2 its variance. Each of these parameters is country specific. The estimation procedure uses a nonlinear least squares method, based on an approximated expenditure function. For details on the computation of the expenditure function, we refer the reader to appendix B.

Were the path of credit ceilings observed, we could easily recover the parameters γ and $\widetilde{\beta}$ from a nonlinear least squares regression by minimizing the distance between observed expenditure and predicted expenditure. Since the credit ceiling is unobserved, this method cannot be applied. However, the distribution of the credit ceiling is implied by the parameters θ^2, which we can use to obtain a predicted distribution of expenditure. A natural way is to minimize the distance between observed expenditure and the predicted conditional mean, calculated by simulation.

However, such an objective function produces an inconsistent estimator for a fixed number of simulations. To overcome this problem we follow Laffont, Ossard,

[7]Other attempts along this line are Garcia, Lusardi, and Ng (1995) and Jappelli, Pischke, and Souleles (1995), who use more information to select an unconstrained subsample. The same objection applies, however.

and Vuong (1995) by minimizing the criterion:

$$
\begin{aligned}
l_{T,S}(\theta) &= \frac{1}{T}\sum_{t=1}^{T}\left[\left(\widetilde{e}_t - \frac{1}{S}\sum_{s=1}^{S} e(\widetilde{x}_t, \widetilde{y}_t, R_t, \widetilde{A}_{st}, \theta)\right)^2 \right. \qquad (11)\\
&\left. -\frac{1}{S(S-1)}\sum_{s=1}^{S}\left(e(\widetilde{x}_t, \widetilde{y}_t, R_t, \widetilde{A}_{st}, \theta) - \frac{1}{S}\sum_{s=1}^{S} e(\widetilde{x}_t, \widetilde{y}_t, R_t, \widetilde{A}_{st}, \theta)\right)^2\right].
\end{aligned}
$$

Here S is the number of simulations and $\widetilde{A}_{st}$ is a random draw of the credit ceiling from the normal distribution with mean μ_A and standard deviation σ_A. This criterion contains two terms. The first one is the standard sums of squares of the distance between the observed path of expenditure and the average predicted one. The second term corrects for the inconsistency bias introduced by the random draws of the credit ceiling shocks.

Although introducing the credit ceiling as an unobserved, stochastic, state variable makes our estimation task much more cumbersome, it provides a natural explanation for why the model does not fit the data exactly. Otherwise, we would have to appeal to measurement error in the expenditure series, which seems to us rather unattractive. Moreover, if we were to treat the credit ceiling as constant, we would be unable to explain movements in debt when the constraint is binding. Here, the randomness in $\widetilde{A}_t$ provides a natural explanation.

Since $\widetilde{A}_t$ is not known, we calculate cash-on-hand $\widetilde{x}_t$ as follows: We make an initial guess for μ_A and σ_A. We then draw, from the implied distribution of $\widetilde{A}_t$, a simulated series of the credit ceiling A_{st}. We use this series, in combination with our data on income, debt, and the interest rate, to construct cash-on-hand according to equation (3). We then substitute this series, along with the simulated series for $\widetilde{A}_{st}$, into our objective function (11). This function is then minimized with respect to θ^2 to obtain a new estimate. The process repeats until it converges. Under standard regularity assumptions, the asymptotic distribution of the estimators is normal and root-T consistent, for any fixed S (see Laffont, Ossard, and Vuong (1995)).

The minimization is done country by country using a simplex algorithm. The number of simulations was set to 20, a standard value in this type of study. We did not impose any bound on the credit ceiling or on its variance (except that the variance be positive). The remaining parameters were constrained to remain in certain ranges to prevent the estimation routine from crashing if an abnormal value was reached during the minimization process. Specifically, the curvature of the utility function γ was constrained to lie between 1 and 6 and the discount factor between .44 and 1. If a bound was reached, the estimation was started again with larger bounds. The estimation takes about five hours for each country on a Pentium 133 running GAUSS.

3.2 Monte Carlo Analysis

To evaluate the performance of the second stage of the estimation method on small samples, we first estimate the model with simulated data. We generate 100 series 21 periods in length for the exogenous variables (the interest rate, income, and the credit ceiling). We then compute optimal debt and expenditure implied by the paths. Parameter values are set at levels consistent with other studies, but we do not further defend this parameterization. The curvature of the utility, γ is set at 2, the discount rate $\widetilde{\beta}$ at 0.85, the credit ceiling at 40 per cent of mean GDP, and the variance of the shocks to the credit ceiling is set at 2% of mean GDP. Unobserved shocks to the credit ceiling generate randomness.

Table I: Monte Carlo Results

Parameters	True	5% Perc.	Median	95% Perc.	Mean	S.D.
γ	2	1.97	2.47	4.38	2.67	(0.76)
β	0.85	0.85	0.90	0.93	0.90	(0.02)
μ_A	-40	-47.47	-31.8	-19.58	-32.9	(9.58)
σ_A	2	1.14	4.2	6.52	4.1	(1.46)
DW		0.53	1.48	2.46	1.51	(0.63)
R^2		0.16	0.85	0.95	0.79	(0.22)

Notes: Results obtained on 100 replications for a sample size of 21.

Table I reports summary statistics from the estimation. The results demonstrate, not surprisingly, some small sample bias in the estimate. However, the true values lie in the range of one or two times the standard deviation. The results are closest for the curvature parameter γ and μ_A. The estimated discount factor $\widetilde{\beta}$ and variance of the credit ceiling σ_A^2 are slightly higher than the true values. However, even for these parameters the true values lie within two standard errors. The Durbin-Watson test is skewed to the left, but the correct value is still in the confidence interval. The R^2 is relatively high at 0.79 on average. While the extent of small-sample bias is small, we nevertheless take it into consideration in the next section, where we present the results of estimating the model with our data.

4 An Application to Sovereign Debt

We apply our methodology to sovereign country borrowing in international capital markets during 1973-1993. For reasons discussed in the introduction, there is strong reason to think that these countries faced borrowing constraints.

We first discuss our data. Some preliminary statistical tests confirm the hypotheses that access to credit was indeed constrained. We turn to the estimation of the parameters of our model. We conclude with a discussion of some of their implications.

4.1 The Data and Evidence of Constraints

The countries under study are mainly African and South American countries, but we also have series for some countries in Asia, Europe, and Latin America.[8] These

[8]We chose countries on the basis of the availability of time series on public and private debt and net resource transfers from creditors in the World Bank's *World Debt Tables*. Time series of sufficient length were available for: Algeria, Argentina, Bolivia, Brazil, Burundi, Cameroon,

countries were all classified by the World Bank as moderately or severely indebted low or middle income countries.

We obtain a time series on per capita expenditure and private debt for each country from data on real GDP, real net resource transfers from official sources, real debt to private creditors, and population.[9] Figure 2 presents the ratio of debt to GDP for the period 1973-1993 for the entire group. As indicated by Figure 2, the share of debt in GDP has been substantial, especially during the early eighties, reaching about 30% on average. For some countries in the sample, the ratio can reach 100%. However, as indicated by the hump shape of the graph, most of the debt was repaid by the mid nineties.

To obtain a more precise view on this point, we calculate the percentage of 1980 private and official debt paid during the period 1981-1993, calculated as the present value in 1980 discounting by the London Interbank Offer Rate (LIBOR).[10] Table II presents the results. Patterns for official debt are clearly different from private debt. Private banks have been more successful in recovering their debt for more than 90% of the countries in our sample. On average, about 60% of the private 1980 debt has been repaid, whereas this figure is only 23% in the case of official debt. The percentage is sometimes negative for official debt, indicating that the public debt has increased. Since public debt has not generated large net resource transfers from borrowers, and since these creditors have noncommercial motives for providing net resource transfers to borrowers, we treat net resource transfers from public sources as an exogenous addition to income over GDP. We thus think of our model as applying to borrowing from private sources.

One striking feature of the data is the link between expenditure and current GDP. If we consider a standard permanent income model, assuming perfect credit markets and a quadratic utility function, borrowing should smooth the effects of temporary variations in current income on expenditure. Only shocks to *permanent* income should affect expenditure. Thus, the growth of expenditure in a period should not be correlated with GDP growth that period, correcting for the innovation in permanent income. We examine this hypothesis with an excess sensitivity test.

Central Africa, Chile, Colombia, Congo, Costa Rica, Cote d'Ivoire, Egypt, Ecuador, Ethiopia, Gabon, Ghana, Honduras, Hungary, Indonesia, Malawi, Morocco, Mexico, Nigeria, Peru, Philippines, Poland, Chad, Thailand and Zaire.

[9]Data on population and real GDP are from Heston and Summers (1991). Data on net resource transfers from official sources (both multilateral and bilateral), and debt to private sources are from the World Bank's *World Debt Tables*. Real values of nominal variables were obtained by deflating with the U.S. wholesale or producer price index (depending on the period) taken from the International Monetary Fund's *International Financial Statistics*. For the period 1978-1993 the nominal interest rate is 3-month London Interbank Offer Rate (LIBOR) as reported by the *International Financial Statistics*. For the period 1973-1977 this interest rate is not reported. The 3 month Eurodollar rate is used instead. (For the period 1978-1980 during which both rates were reported they differ by at most .17 per cent with little difference in mean.) To obtain a real interest rate the interest rate series was adjusted by the subsequent change in the U.S. dollar wholesale or producer price index.

[10]Similar calculations can be found in Cohen (1992). While the interest rates charged on these loans were marked up over LIBOR, we treat the mark-up as a risk premium to compensate for anticipated nonpayment. Hence we interpret LIBOR itself as the actual cost of the loan.

Table II: Percentage of 1980 Debt Paid during 1981-1993

Country	Public Debt	Private Debt	Country	Public Debt	Private Debt
Algeria	0.62	0.63	Ethiopia	0.12	-0.16
Argentina	-0.20	0.45	Gabon	-0.23	0.73
Bolivia	0.40	0.82	Ghana	0.51	0.62
Brazil	0.24	0.68	Honduras	0.26	0.73
Burundi	-0.31	0.68	Hungary	-0.1	0.32
Cameroon	0.40	0.68	Indonesia	0.28	0.44
Central Afr.	-0.18	0.78	Malawi	0.42	0.82
Chad	0.39	0.83	Mexico	0.03	0.62
Chile	0.20	0.70	Morocco	0.27	0.68
Colombia	0.27	0.36	Peru	0.52	0.55
Congo	0.12	0.64	Philippine	-0.04	0.61
Costa Rica	0.40	0.77	Senegal	0.23	0.77
Cote d'Ivoire	0.07	0.72	Uruguay	0.37	0.47
Ecuador	0.39	0.58	Venezuela	0.35	0.57
Egypt	0.58	0.31			

We estimate:

$$\Delta\widetilde{e}_t = \pi\Delta\widetilde{y}_t + \phi u_t + v_t \tag{12}$$

where $\widetilde{e}_t$ are detrended expenditures, u_t is the innovation in (unobserved) permanent income, and $\widetilde{y}_t$ is detrended income, i.e., GDP in plus net resource transfers from official creditors, all in period t. We construct a proxy for innovations in income by fitting a AR(1) to the $\widetilde{y}_t$ series.[11] Under the null hypothesis of perfect capital markets and quadratic preferences, the coefficient π should be zero. These tests have often been used to test for liquidity constraints (e.g., by Flavin (1985), Campbell and Mankiw (1989), and Lewis (1997)). The results are presented in Table III. For most of the countries in the sample (over 80%), we cannot reject the hypothesis that π is strictly positive at the 5% level.[12] Moreover, given that these countries are relatively poor, and given the history of the debt crisis in the mid eighties, the result is hardly surprising. The purpose of our subsequent analysis is to quantify the borrowing constraints facing sovereign borrowers.

4.2 Estimation

We estimate the model for a set of debtor countries using, for each country, three time series: (i) income (real GDP per capita augmented by real net resource transfers from official sources), (ii) real debt to private creditors, and (iii) the real world interest rate. The series are annual aggregate data for the years 1973 through 1993. We proceed in two broad steps. First, we estimate the structural parameters of the model, country by country, on the time dimension of our dataset, using the procedure described in section 3. Second, we explore the determinants of the estimated credit ceilings using the cross-section dimension of our dataset. As discussed above, the

[11]The results are robust for a higher order AR process.

[12]In fact, this test does not discriminate well between nonseparabilities in the budget constraint, e.g. liquidity constraints, and in the utility function, e.g., habit persistence. See Deaton (1991). In our approach here, however, we explore the first explanation, as it seems the more natural one in the context of sovereign borrowing.

Table III: Excess Sensitivity Test

Country	π	t stat	Country	π	t stat
Algeria	1.362	(3.84)	Ethiopia	1.067	(8.44)
Argentina	0.962	(24.17)	Gabon	1.223	(7.57)
Bolivia	1.050	(3.75)	Ghana	1.000	(116.82)
Brazil	1.020	(6.43)	Honduras	1.090	(10.78)
Burundi	1.031	(16.13)	Hungary	1.936	(1.60)
Cameroon	1.175	(10.41)	Indonesia	0.928	(7.28)
Central Africa	1.018	(14.49)	Malawi	1.273	(13.20)
Chad	1.061	(21.32)	Mexico	0.751	(4.25)
Chile	1.052	(6.40)	Morocco	1.446	(6.28)
Colombia	1.084	(4.92)	Peru	0.978	(11.73)
Congo	1.236	(1.69)	Philippine	0.818	(1.63)
Costa Rica	1.099	(5.08)	Senegal	1.172	(10.63)
Cote d'ivoire	1.650	(3.17)	Uruguay	0.882	(4.87)
Ecuador	1.072	(4.96)	Venezuela	1.235	(4.62)
Egypt	1.052	(12.56)			

Notes: π is the coefficient in front of GDP growth. Innovation to permanent income computed by fitting an AR(1). The results are robust for higher AR processes.

estimation of the structural model itself takes two steps: (i) estimating the processes of the exogenous variables in order to detrend them; (ii) estimating the remaining parameters of the structural model using detrended data.

4.2.1 Adjusting for Trends and Autocorrelation

We begin by estimating, for each country, the exponential trend α, the variance of the innovation of income σ_Y^2, and the autocorrelation of income ρ_Y. Although the estimation in two steps is less efficient, the computing time is reduced by a substantial amount as the number of parameters to estimate in the second step decreases from eight to four. Table IV presents the results of this first step.

Note that during this period average growth in 16 of these countries was actually negative, and in six (such as Chad and Venezuela) it is significantly negative. In only five is it significantly positive. Not surprisingly, income in all our countries is highly autocorrelated. We also tested assumption 2 by testing for serial correlation in the residuals of the income equation. At the 5% level, we cannot reject the null hypothesis of zero serial correlation for 26 countries of the sample. For the 5 remaining countries, the t-statistic is never higher than 2.7. Once this preliminary estimation is performed, we remove the trend from the series of expenditure, GDP and debt.

Turning to the real interest rate series, (LIBOR minus the percentage change in the U.S. dollar producer price index) we estimate μ_R at 0.12, ρ_R at 0.72 and σ_R^2 at 0.04.

Table IV: GDP Characteristics

Country	μ_Y	σ_Y	ρ_Y	α	Country	μ_Y	σ_Y	ρ_Y	α
Algeria	2383.5	274.6	0.80	0.0011	Ethiopia	165.7	20.4	0.78	-0.0081
			(0.13)	(0.009)				(0.23)	(0.006)
Argentina	5080.5	1811.0	0.44	-0.0024	Gabon	5039.1	990.8	0.49	-0.0225
			(0.22)	(0.014)				(0.17)	(0.008)
Bolivia	1008.7	101.7	0.67	-0.0160	Ghana	1123.7	988.2	0.75	-0.0728
			(0.12)	(0.006)				(0.15)	(0.020)
Brazil	2427.4	242.9	0.70	0.0012	Honduras	867.3	129.9	0.70	-0.0106
			(0.17)	(0.006)				(0.18)	(0.007)
Burundi	256.3	25.4	0.83	-0.0033	Hungary	2602.5	130.7	0.71	0.0205
			(0.16)	(0.006)				(0.17)	(0.003)
Cameroon	932.9	87.3	0.70	0.0234	Indonesia	590.6	45.2	0.66	0.0191
			(0.19)	(0.006)				(0.15)	(0.006)
Central Afr.	406.4	38.6	0.62	0.0144	Malawi	230.9	23.6	0.69	-0.0132
			(0.19)	(0.005)				(0.17)	(0.004)
Chad	233.8	60.5	0.66	-0.0266	Mexico	2780.1	428.9	0.68	0.0061
			(0.19)	(0.013)				(0.19)	(0.008)
Chile	2327.9	383.3	0.69	0.0070	Morocco	994.8	114.9	0.77	-0.0061
			(0.17)	(0.009)				(0.16)	(0.007)
Colombia	1378.2	82.2	0.85	0.0013	Peru	1607.8	263.6	0.41	0.0073
			(0.11)	(0.005)				(0.21)	(0.007)
Congo	1179.0	123.1	0.70	0.0048	Philippine	765.4	53.4	0.79	-0.0043
			(0.18)	(0.006)				(0.14)	(0.004)
Costa Rica	2088.1	364.6	0.65	-0.0058	Senegal	675.5	88.4	0.67	-0.0037
			(0.19)	(0.008)				(0.19)	(0.008)
Cote d'Ivoire	1152.5	197.3	0.75	-0.0245	Uruguay	2982.0	534.1	0.78	0.0036
			(0.15)	(0.009)				(0.16)	(0.010)
Ecuador	1435.8	195.0	0.75	-0.0102	Venezuela	3941.9	645.5	0.54	-0.0318
			(0.12)	(0.008)				(0.17)	(0.006)
Egypt	1046.6	168.1	0.78	0.0229					
			(0.16)	(0.012)					

Notes: standard errors in parenthesis.

4.2.2 Results

Having detrended the data and obtained estimates of the variance and autocorrelation of the observed exogenous state variables, we now estimate, for each country, the 4×1 vector of parameters $\theta^2 = [\gamma, \widetilde{\beta}, \mu_A, \sigma_A]$. Table V reports the results. Heteroscedastic corrected standard errors are reported in parentheses.

Given how simple our model is, it performs surprisingly well. Column 5 in Table V reports the R^2 of the regression. For most of the countries, the R^2 is high, around .8 or .9. The model seems to capture most of the variability in the data. The fit is poor for Congo, Hungary and Ghana. For the last country, the computation of the equilibrium was problematic as the standard deviation of the income process is almost as high as the mean.

The Durbin-Watson test shows significant serial correlation in the residuals for almost all our estimations. We can put forth two reasons for this serial correlation. First, this might indicate that we fail to take into account some other important state variable. One obvious candidate would be investment. Omission of such a variable would generate serial correlation.[13] A second reason could be the effect of small sample bias on the Durbin-Watson statistic as pointed out in the Monte Carlo exercise.

[13] In future work we plan to endogenize the investment decision.

For most countries the estimation procedure yields reasonable parameter estimates. The estimated discount factor $\widetilde{\beta}$ usually implies a (growth-adjusted) discount rate between 12 and 25 per cent. Recall that our model includes investment in expenditure, so that the discount factor incorporates a desire to invest as well as to consume. The estimated curvature of the period utility function γ is usually greater than one but rarely less than 3. This range is consistent with that obtained in many other studies.[14] Columns three and four reports the mean and the standard error of the credit ceilings. The parameter μ_A is always negative, which constitutes a first check of the results. The ratio of the estimated per capita credit ceiling to per capita income ranges from 1% (Burundi) to 73% (Cote d'Ivoire), with an average around 27%. This ratio is the lowest for the poor African countries. The standard deviation of the credit ceiling is positively related to the mean of the credit ceiling, ranging from 2% for Venezuela to 180% for Burundi. The average ratio of standard deviation of the credit ceiling to its mean is 18%.

4.3 Determinants of Credit Constraints

Figure 3 plots the natural logarithm of our estimate of the credit ceiling against mean income in our sample of countries. Observations are labelled according to the code by the country's name in table V. The relationship does suggest a strong positive association between income and access to credit that is fairly uniform across the countries in our sample.

The theoretical literature on sovereign debt suggests that a country's willingness to service debt depends upon the cost of the penalties that would be imposed upon it should it default. Potential penalties include a cutoff of trade, loss of access to capital markets, and a reduction in net transfers from official sources.[15] The literature has also suggested that banks would be more willing to lend to countries that invest a large fraction of GDP and to countries whose governments spend a smaller fraction of GDP, particularly military spending. Finally, a country's population and access to oil might affect how much credit lenders would be willing to extend.[16]

To explore these hypotheses we regress our estimates of the mean of credit ceiling and its variance on country characteristics that reflect these country characteristics. Table VI reports the results of regressing $ln(-\mu_A)$ and $ln(\sigma_A)$ on log of mean income per capita (GDP), the log of the variance of GDP as a share in total income, log of exports plus imports each as a share of GDP (OPEN) and the log of the share of domestic investment in the GDP. The regression is instrumented by variables correlated with GDP or openness, but not with credit ceilings. We used the mean of human capital, the log of GDP in 1960, the distance between the main city and the sea, as well as the area of the country.

We first consider the results for the mean of the credit ceiling μ_A. Three variables, GDP, openness and domestic investment are significant at the 5% level in explaining the mean of the credit ceilings. The variance of the GDP appears non significant. Theory says much less about the determinants of the variance of the credit ceiling.

[14] See the discussion, for example, in Prescott (1986).

[15] See, for example, the discussions in , Gersovitz (1983) and Bulow and Rogoff (1989).

[16] See, for example, Cohen and Sachs (1986).

We find that it is positively related to the mean of GDP and negatively to the share of domestic investment. The other variables are statistically insignificant.

The regression for μ_A implies that a country with 10 per cent higher income has access to 16% more in credit. A rise of 10% in openness increases the credit ceiling by around 8%. Similarly, a rise in 10% in the share of domestic investment increases the credit ceiling by the same amount.

We have also computed the probability of a binding constraint for each country in each year. Using the computed $x^*(\widetilde{y}^i_t, R_t, \widetilde{A}^i_{st}, \theta^i)$ for each date t, each simulation s, and each country i, the liquidity constraint is binding whenever $\widetilde{x}^i_{st} < x^*(\widetilde{y}^i_t, R_t, \widetilde{A}^i_{st}, \theta^i)$. We present in Figure 5 the percentage of liquidity constrained countries in the sample against time. This percentage ranges from 30% to 80%, and is humped shape. Consistent with the history of the debt crisis, the constraint is most binding in the period 1980 through 1990.

4.4 An Example: The Case of Brazil

It is instructive to look at the path of debt and cash-on-hand for some countries. We focus here on Brazil, but we could have selected other countries as well. Figure 4 plots the paths of our estimates of cash-on-hand $\widetilde{x}_t$ and the cut-off limit x^*_t , as well as the path of the mean credit ceiling (along with a two-standard-deviation confidence band) and the path of actual debt. We also report the 95% confidence bands, based on realizations of $\widetilde{A}_{st}$. Remember that a country is constrained whenever cash-on-hand $\widetilde{x}_t$ falls below the cut-off limit x^*_t. During the period 1973-1981 cash-on-hand is on average higher than the cut-off limit, so the country is not estimated to be constrained, even though debt is close to the mean of the credit ceiling in 1978. From 1982 to 1988, cash-on-hand is below the cut-off limit at which the constraint is binding. This is the period in which debt is at its highest, somewhat higher than the estimated mean of the credit ceiling. From 1989 to 1990, debt is decreasing, and the constraint is not binding. In 1991-92, the constraint appears binding, and debt continues to decreased as a consequence of the Collor reforms which cut expenditure. The model interprets this drop in expenditure as the consequence of a tightening of the credit constraint.

Note that in 1978 and in 1982 Brazil has similar debt levels, but we estimate different probabilities of being constrained. This difference is explained by the randomness we allowed in the credit ceilings. The model interprets the lower level of expenditure in the second period as indicative of a binding credit constraint.

4.5 Welfare Analysis

In this section, we use our parameter estimates to calculate the effect of an increase of the credit ceiling on the flow of utility. To do so we first calculate the discounted flow of utility generated under the values of the parameters presented in Table V as a baseline value. We then recalculate the flow of utility with μ_A increased by some amount, holding the other parameters at their estimated value. We assume that the increase in the credit ceiling is a surprise for the country.

We can perform this experiment either *ex post* or *ex ante.* In the first case we ask, given the path of income and the interest rate that subsequently occurred, how much better off would the country have been if it could have borrowed more. Given the realizations, it could happen, for example, that a country might, after the fact, been better off if it had not incurred so much debt. In the second case we ask how much better off would the country have been if it could have borrowed more given the expected path of income and the interest rate at the beginning of the period. From this perspective it must be the case that a country is always better off by having greater access to credit.

From an *ex post* perspective, our results imply that, on average, a ten per cent increase in the credit ceiling would have raised welfare on average by a modest .1%. The effect is negative for Congo and Venezuela. The fit for the first country is poor anyway. For Venezuela, the liquidity constraint rarely binds, so the increase of the credit ceiling has little opportunity to raise welfare.

From an *ex ante* perspective the results are very different. Table VII presents the percentage increase in expected discounted utility resulting from a ten per cent increase in the credit ceiling from the perspective of the beginning of the period. An increased credit ceiling allows the country more smoothing possibilities. While, for some countries (e.g., Chad and Burundi), the effect is trivial, for others (e.g., Congo and Ghana), expected welfare would have been around 7 per cent higher.

The difference between the effects *ex ante* and *ex post* imply that history dealt out realizations of income, the interest rate, and the credit ceiling that made access to loans much less rewarding than the parameters of these processes would have suggested. A particular culprit is the real interest rate, which rose substantially in the 1980s after these countries had accumulated most of their debt (at variable rates).

5 Conclusion

We have developed a methodology for estimating the optimal borrowing behavior by an intertemporal utility-maximizing agent subject to credit constraints. To this end, we characterize the stochastic rational expectation equilibrium, which gives optimal expenditure and net transfer given the state variables.

We allow four state variables: cash-on-hand, income, the interest rate, and a stochastic credit ceiling, which we do not observe. We then estimate the preference parameters as well as the parameters of the distribution of the credit ceilings. This constitutes a major contribution of this paper, as the previous literature examining the potential for credit rationing focused on tests of liquidity constraints rather than the estimation of their level and their determinants.

We apply the methodology to estimate borrowing by a group of sovereign debtors during 1973-1993, a group of borrowers who are very likely to have faced severe borrowing constraints. We find that introducing credit ceiling vastly improves the ability of the Ramsey model to explain expenditure. Moreover, the procedure yields reasonable estimates of the discount factor, elasticity of marginal utility of expenditure, and the credit ceiling for most of the countries in our sample. Our estimated credit ceilings rise quite regularly with income across the countries of our sample, and are positively associated with trade, in line with several theoretical models.

We regard our methodology here as a first step in developing a framework for estimating the behavior of intertemporally maximizing agents subject to liquidity constraints. There are a number potentially useful directions that future research could take. A particular limitation of what we do here is the exclusion of capital accumulation. Introducing capital requires the specification of a more complex model and the use of additional, and often less reliable, data. Nevertheless, since allowing for capital accumulation would allow us to examine the potential effects of credit constraints on growth, the benefits seem worth the cost.

A Proof of Proposition 4

Our proof follows Chambers and Bailey (1994), modified to accommodate a time-varying interest factor R. Where relevant, all variables are detrended. We first prove the following results:

Definition 1 *The mapping $T : g \to f$ defined for each possible realization of the state variables $s = \{y, R, A)$ is:*

$$f(x,s) = \max\left[\beta R \int_S g\{R[x - \lambda^{-1}(f(x,s))] + y' - A' + RA, s'\}dF(s'|s), \lambda(x)\right]$$

(Here f and g correspond to the marginal utility of expenditure $e(x,s)$, that is, $e(x,s)^{-\gamma} = \lambda[e(x,s)]$.) The stationary rational expectations equilibrium solves $f = Tf$.

The function g is chosen to belong to the set $\mathbf{G}$, defined next:

Definition 2 $\mathbf{G}$ *is the space of functions $g(x,s)$ such that:*

(i) The domain Λ of $g(x,s)$ is $\mathbf{R}^+ x \mathbf{R}^+ x \mathbf{R}^+ x \mathbf{R}$. (ii) $g(x,s)$ is continuous, nonnegative, and nonincreasing in x. (iii) $g(0,.)$ has an upper bound $\overline{\lambda}$.

Definition 3 *The mapping $T_N : g \to f$ is $f = T^N g$.*

Lemma 1 *T_N maps $\mathbf{G}$ into itself.*

Definition 4 *The function G is:*

$$G(q,x,s) = \beta R \int_S g\{R[x - \lambda^{-1}(q)] + y' - A' + RA, s'\}dF(s'|s). \tag{13}$$

for $(x,s) \in \Lambda$ and $q \leq \overline{\lambda}$.

We first show that G is continuous. Choose a sequence $(q_n, x_n, s_n) \to (q, x, s)$. By the triangle inequality:

$$\begin{aligned} G(q,x,s) - G(q_n,x_n,s_n)| &\leq |G(q,x,s) - G(q,x,s_n)| + |G(q,x,s_n) - G(q_n,x_n,s_n)| \\ &\leq |G(q,x,s) - G(q,x,s_n)| + H_n \end{aligned}$$

where

$$H_n = \beta \int_S R_n \quad [|g\{R_n[x - \lambda^{-1}(q)] + y' - A' + R_n A_n, s'\} \\ -g\{R_n[x_n - \lambda^{-1}(q_n)] + y' - A' + R_n A_n, s'\}|] \\ dF(s'|s_n),$$

where the second inequality results from the fact that $\int |f(x)|\, dx \geq \left|\int f(x)dx\right|$. The first term on the RHS of the inequality goes to zero since g is bounded and continuous. The continuity of g also ensures that H_n also goes to zero. Hence G is continuous. Next note that since g is bounded above by $\overline{\lambda}$, G has an upper bound $\beta\overline{R\lambda}$. Since g is nonnegative so is G. Since λ^{-1} is decreasing in q, $x-\lambda^{-1}$ is increasing in q. Since, by assumption, g is decreasing in x, G is decreasing in q and x. Hence $G-q$ is strictly decreasing in q. Since G has an upper bound, $G-q$ is negative for q sufficiently large. For any $s \in S$, and x, $q = f(x,s)$ solves:

$$\max[G(q,x,s) - q, \lambda(x) - q] = 0$$

If $G[\lambda(x), x, s] - \lambda(x) \leq 0$ then, since G is monotonically decreasing in q, $f(x,s) = \lambda(x)$ while if $G[\lambda(x), x, s] - \lambda(x) \geq 0$ then $f(x,s) = q$ such that $G[f(x,s), x, s] - f(x,s) = 0$. Since G and λ are continuous and decreasing in x, and G is decreasing in q, f is continuous and nonincreasing in x. Since λ and G are nonnegative, so is f. The monotonicity of f in x implies that it is at a maximum when $x = \underline{x} = \overline{R}\underline{A} + \underline{y} - \overline{A}$, the lowest possible realization of x. It remains to show that $f(\underline{x}, .) \leq \overline{\lambda}$. If $f(\underline{x}, s) = \lambda(\underline{x})$ then boundedness follows immediately. Alternatively, if $f(\underline{x}, s) = G[f(\underline{x}, s), \underline{x}, s]$ then:

$$f(\underline{x}, s) < \beta^N E\left[\left(\prod_{i=1}^{N+1} R_{t+i-1}\lambda_{t+N+1}\right)|s\right] \leq \beta^N E\left[\prod_{i=1}^{N+1} R_{t+i-1}|s\right]\overline{\lambda} \leq \overline{\lambda}$$

The next-to-the last inequality follows from the fact that, for any pair of random variables a and b with supports A and B, where b has an upper bound $\overline{b}$:

$$\int_A\int_B ab df(a)dg(b|a) \leq \int_A a\overline{b}df(a).$$

Thus $f = Tg$. Since T maps the space of functions $\mathbf{G}$ into itself so does T_N.$\|$

Lemma 2 *T_N is a contraction mapping.*

We establish Blackwell's two sufficient conditions (See, e.g., Stokey and Lucas, 1989). 1. For any pair g_0, $g_1 \in \mathbf{G}$ such that

$$g_1(x,s) \leq g_0(x,s) \qquad \forall x, y \in \Lambda$$

T_N preserves the inequality. Note that:

$$\begin{aligned} G_1(q,x,s) &= \beta R\int_S g_1\{R[x - \lambda^{-1}(q)] + y' - A' + RA, s'\}dF(s'|s) \\ &\leq G_0(q,x,s) = \beta R\int_S g_0\{R[x - \lambda^{-1}(q)] + y' - A' + RA, s'\}dF(s'|s) \end{aligned}$$

For any q and $(x,s) \in \Lambda$, let q_0 solve $G_0(q,x,s) = 0$ if there is a solution. Since $G_1(q,x,s) \leq G_0(q,x,s)$, $G_1(q_0,x,s) - q_0 \leq 0$. Hence the solution q_1 to $G_1(q,x,s) - q_1$, if it exists, must satisfy: $q_1 \leq q_0$ (since we established above that $G - q$ was strictly decreasing in q.) If $G_0[\lambda(x), x, s] - \lambda(x) \leq 0$ then, since G is monotonically decreasing in q, $q_0 = \lambda(x)$. But since $G_1 \leq G_0$, then $q_1 = \lambda(x)$ as well, so that $q_1 = q_0$. Finally, if q_0 solves $G_0(q,x,s) = q$ while $q_1 = \lambda(x)$ then $G_0[\lambda(x), x, s] - \lambda(x) \geq 0$, so that $q_0 \geq q_1$. Hence $T(g_1) \leq T(g_0)$. Hence $T_N(g_1) \leq T_N(g_0)$. 2. For any $g \in \mathbf{G}$ and any positive scalar a, $T_N(g+a) \leq T_N(g) + ka$ for some $k < 1$

$$\begin{aligned}
T(g+a) &= \max\{G[g(x.s), x, s] + a, \lambda(x)\} \\
&\leq \max\{G[g(x.s), x, s], \lambda(x)\} + \beta R a \\
&= T(g) + \beta R a
\end{aligned}$$

Hence:

$$T_N(g+a) \leq T_N(g) + \beta^N \overline{R}_N a \leq T_N(g) + ka/$$

||

Lemma 3 *The space* $\mathbf{G}$ *is complete.*

See Chambers and Bailey (1994), Technical Appendix, Lemma 3.

Lemma 4 *There exists a unique marginal utility function* $f : \Lambda \to \mathbf{R}$ *such that* $f(.,.)$ *is continuous, nonnegative, nonincreasing in* x*, that satisfies:*

$$f(x,s) = \max\left[\beta R \int_S f\{R[x - \lambda^{-1}(f(x,s))] + y' - A' + RA, s'\} dF(s'|s), \lambda(x)\right].$$

Choose any $g_0 \in \mathbf{G}$. Since T_N is a contraction the sequence

$$g_0, g_1 = T_N g_0, g_2 = T_N g_1, ...$$

converges uniformly to a unique equilibrium $f \in \mathbf{G}$. ||

Lemma 5 *If* $T_N : \mathbf{G} \to \mathbf{G}$ *is a contraction mapping with* $f = T_N f$ *then if* $\mathbf{G}' \subseteq \mathbf{G}$ *is closed, and if* $T_N(\mathbf{G}') \subseteq \mathbf{G}'$ *then* $f \in \mathbf{G}'$.

See Lucas and Stokey, page 52.

Lemma 6 f *is decreasing in* y *and* A *and increasing in* R

See Chambers and Bailey, Technical Appendix, Theorem 3, noting that $R \geq 0$ while $A \leq 0$.

Definition 5 *A cutoff limit* $x^*(s)$ *solves:*

$$G[\lambda(x^*(s)), x^*(s), s] = \lambda(x^*(s)). \tag{14}$$

Lemma 7 *There is a unique cutoff limit is unique such that* $f(x,s) = \lambda(x)$ *for* $x < x^*(s)$ *while* $f(x,s) \geq \lambda(x)$ *for* $x \geq x^*(s)$.

Note that, from the definitions of G and λ, we can define:

$$H(s) = G[\lambda(x), x, s].$$

Since H is independent of x while λ is monotone in x, there is at most one x that solves 14. The proof of lemma 2 established that, if $H(s) \leq \lambda(x)$ then $f(x, s) = \lambda(x)$ while if $H(s) > \lambda(x)$ then $f(x, s) \geq \lambda(x)$. Since $\lambda(x)$ is decreasing in x, then, the result follows.∥

Lemma 8 *The cutoff limit x^* is increasing in y' and A and decreasing in R.*

Note that H is decreasing in y' and A and increasing in R. Since λ is monotone in s the result follows. The theorem follows from the properties of f and observing that $e(x, s) = \lambda^{-1}[f(x, s)]$. ∥

B Computation of the Expenditure Function

Numerically, there are several ways to derive the optimal expenditure function.[17] Deaton (1991) chooses a backward recursive (nonparametric) method to find the fixed-point using the first-order condition. The main drawback of this method is that it is difficult to accommodate more than two state variables, and we have three, x, R, and A. The approach taken here follows the orthogonal collocation method proposed by Judd (1992). The idea is to approximate the expenditure function by a basis of functions such as Chebyshev polynomials. As these polynomials are orthogonal polynomials we can use higher order polynomials and avoid multicolinearity. The expenditure function is a non linear function with a kink at $x = x^*$. Approximating it on the whole cash-on-hand space is therefore difficult. We take advantage of the properties of the expenditure function in the constrained regime, where the function is equal to the 45 degree line. We then approximate the unconstrained part of the expenditure function, as well as the cut-off limit x^*.

$$\begin{aligned}
e(x, y, R, A) &= x && \text{for} \quad x < x^*(y, R, A) \\
e(x, y, R, A) &= \sum_{i,j,l,n=1}^{p} \omega^1_{i,j,l,n} T_i(x) T_j(y) T_l(R) T_n(A) && \text{for} \quad x > x^*(y, R, A) \\
x^*(y, R, A) &= \sum_{i,j,l=1}^{p} \omega^2_{i,j,l} T_i(y) T_j(R) T_l(A)
\end{aligned}$$

where $T_i(.)$ is a Chebyshev polynomial of order i and $\omega^1_{i,j,l,n}$ and $\omega^2_{i,j,l,n}$ are auxiliary parameters. The approximation is done by finding the matrix $\omega = \{\omega^1_{i,j,l,n}, \omega^2_{i,j,l,n}\}$ such that the first-order condition holds for each set of points (x, y, R, A). This method proves to be reliable, fast enough to be incorporated in an estimation routine, and can accommodate several state variables. We limit the number of interaction terms by constraining some $\omega^k_{i,j,l,n}$ to zero. For the expenditure function, we allow interactions between cash-on-hand and lagged GDP, between cash-on-hand and the interest rate and between cash-on-hand and the credit ceilings. We allow

[17] Most of the numerical methods are summarized in Rust (1995).

polynomials up to the fourth order. For the cut-off limit, we used second order polynomials and no interaction terms between GDP, the interest rate and the credit ceiling.

$$\begin{cases} e(x,y,R,A) = x \qquad \text{if} \quad x < x^*(y,R,A) \\ (x,y,R,A) = \sum_{i=1}^{4}\sum_{j=1}^{4}\omega_{ij}^1 T_i(x)T_j(y) + \sum_{i=1}^{4}\sum_{j=1}^{4}\omega_{i,j}^2 T_i(x)T_j(R) + \sum_{i=1}^{4}\sum_{j=1}^{4}\omega_{i,j}^3 T_i(x)T_j(A) \\ x^*(y,R,A) = \sum_{i=1}^{2}\omega_i^4 T_i(y) + \sum_{i=1}^{2}\omega_i^5 T_i(R) + \sum_{i=1}^{2}\omega_i^6 T_i(A) \end{cases}$$

The expenditure function as well as the cut off function are computed jointly by estimating ω such that the left hand side of equation (6) is as close as possible to the right hand side, by non linear least squares. The function was minimized for values of x, y, R and A which are zeros of the Chebyshev polynomials, which are the most efficient values. The convergence criterion was set such that the maximal error over any points in the grid was lower than .8%. The method was checked against the method used in Deaton (1991) and the error was negligible. The conditional expectation was computed by discretizing the income and interest space by using the quadrature method exposed in Tauchen and Hussey (1991).

Table V: Estimation Results

Country	γ	β	μ_A	σ_A	R^2 [a]	DW [b]
Algeria (AL)	1.89	0.85	-890.90	26.68	0.75	0.54
	(0.43)	(0.07)	(24.74)	(13.92)		
Argentina (AR)	1.14	0.45	-1135.79	490.71	0.97	0.89
	(0.49)	(0.41)	(24.03)	(10.02)		
Bolivia (BO)	1.82	0.78	-470.60	12.15	0.33	0.33
	(1.03)	(0.08)	(25.63)	(4.38)		
Brazil (BR)	1.85	0.81	-585.75	30.47	0.93	1.21
	(0.23)	(0.04)	(8.80)	(1.28)		
Burundi (IN)	1.51	0.63	-2.68	4.80	1.00	0.83
	(0.08)	(0.01)	(0.04)	(0.09)		
Cameroon (BU)	1.83	0.86	-75.72	9.62	0.94	0.74
	(0.21)	(0.08)	(2.58)	(0.76)		
Central Africa (MX)	1.89	0.86	-16.98	5.31	0.93	0.53
	(0.14)	(0.11)	(0.97)	(0.48)		
Chad (MO)	1.16	0.46	-10.87	7.26	0.99	0.86
	(0.00)	(0.00)	(0.03)	(0.00)		
Chile (CH)	2.18	0.69	-963.31	59.20	0.65	0.35
	(0.38)	(0.05)	(31.61)	(20.46)		
Colombia (CO)	1.81	0.74	-196.33	22.51	0.89	0.31
	(0.07)	(0.01)	(4.99)	(2.45)		
Congo (CR)	4.82	0.91	-722.87	18.08	0.07	0.84
	(0.92)	(0.36)	(109.85)	(33.61)		
Costarica (EC)	1.77	0.74	-941.57	32.97	0.75	0.61
	(0.26)	(0.02)	(15.96)	(2.92)		
Cote d'ivoire (CA)	1.57	0.62	-852.14	23.79	0.69	1.08
	(1.11)	(0.25)	(10.40)	(5.69)		
Ecuador (CG)	1.64	0.69	-708.11	19.55	0.79	0.57
	(0.83)	(0.07)	(10.78)	(4.67)		
Egypt (MA)	1.36	0.64	-80.35	14.98	0.97	0.42
	(0.54)	(0.06)	(2.42)	(1.32)		
Ethiopia (PH)	1.52	0.63	-6.46	2.30	0.97	0.57
	(0.31)	(0.04)	(0.26)	(0.15)		
Gabon (EG)	1.59	0.67	-2090.97	64.15	0.50	0.62
	(2.25)	(0.34)	(160.20)	(58.33)		
Ghana (PE)	1.98	0.85	-90.47	11.37	-0.49	0.20
	(7.10)	(3.66)	(3914.61)	(273.90)		
Honduras (SN)	1.55	0.66	-141.29	13.73	0.92	0.37
	(0.29)	(0.04)	(3.13)	(1.39)		
Hungary (ET)	2.10	0.88	-828.07	31.88	-0.87	0.83
	(0.46)	(0.48)	(18.46)	(9.69)		
Indonesia (UR)	1.92	0.84	-87.91	6.50	0.95	1.43
	(0.12)	(0.05)	(0.73)	(0.46)		
Malawi (VE)	1.56	0.70	-24.33	3.48	0.79	0.40
	(0.34)	(0.03)	(1.01)	(0.51)		
Mexico (CI)	1.72	0.73	-801.12	42.92	0.81	0.81
	(0.44)	(0.01)	(19.54)	(6.68)		
Morocco (CE)	1.66	0.69	-236.16	15.27	0.90	0.93
	(0.24)	(0.02)	(2.29)	(1.54)		
Peru (CH)	2.02	0.81	-373.05	19.82	0.78	1.09
	(0.21)	(0.14)	(8.40)	(4.43)		
Philippine (GB)	1.77	0.80	-229.29	8.98	0.64	1.04
	(0.69)	(0.05)	(11.19)	(2.48)		
Senegal (GH)	1.63	0.67	-65.73	13.63	0.94	0.66
	(0.21)	(0.03)	(1.53)	(1.30)		
Uruguay (HG)	1.50	0.56	-647.92	61.74	0.93	0.70
	(0.48)	(0.20)	(12.65)	(7.62)		
Venezuela (HO)	1.74	0.61	-2771.28	66.11	0.67	0.95
	(2.67)	(0.95)	(50.10)	(30.04)		

Notes: Asymptotic heteroskedastic corrected standard errors in parenthesis.

[a] Percentage of variance explained by the model. (computed as $1 - l_{ST}(\theta^*)/\mathrm{var}(e_t)$).

[b] Durbin Watson statistic, computed as $\sum_t (err_t - err_{t-1})^2 / \sum_t (err_t)^2$, with $err_t = e_t - 1/S \sum_s e(x_t, y_{t-1}, r_t, A_{st})$.

Table VI: Determinant of Credit Ceilings

Variable	ln μ_A	t-stat	ln σ_A	t-stat
lnGDP	1.595	7.9	1.35	7.1
lnOpen	.844	2.4	-.118	-0.5
lnVGDP	-.261	-1.0	-.084	-0.5
lnDomInv	.757	3.0	-.429	-1.9
constant	-11.40	-4.5	-5.18	-5.0

Notes: $R^2 = 0.90$ for ln μ_A and $R^2 = 0.88$ for ln σ_A. Number of observation: 29. Heteroscedastic standard errors were computed. Instruments: Log GDP in 1960, mean human capital, distance to sea, area of country.

Table VII: Percentage Increase of Welfare after a 10% Increase in Credit Ceiling

Country	% Increase	Country	% Increase
Algeria (AL)	2.27	Hungary (ET)	2.39
Argentina (AR)	0.28	Philippine (GB)	1.64
Bolivia (BO)	2.42	Senegal (GH)	0.53
Brazil (BR)	1.54	Uruguay (HG)	0.87
Cameroon (BU)	0.57	Venezuela (HO)	2.90
Congo (CR)	7.68	Burundi (IN)	0.05
Cote d'ivoire (CA)	2.30	Central Africa (MX)	0.33
Ecuador (CG)	1.98	Chad (MO)	0.00
Mexico (CI)	1.55	Egypt (MA)	0.25
Morocco (CE)	1.18	Ethiopia (PH)	0.19
Peru (CH)	1.83	Ghana (PE)	6.94
Chile (CH)	3.34	Honduras (SN)	0.75
Colombia (CO)	0.93	Indonesia (UR)	1.10
Costarica (EC)	2.29	Malawi (VE)	0.50
Gabon (EG)	1.68		

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Figure 1: Consumption Function e=e(x,y,r,A)

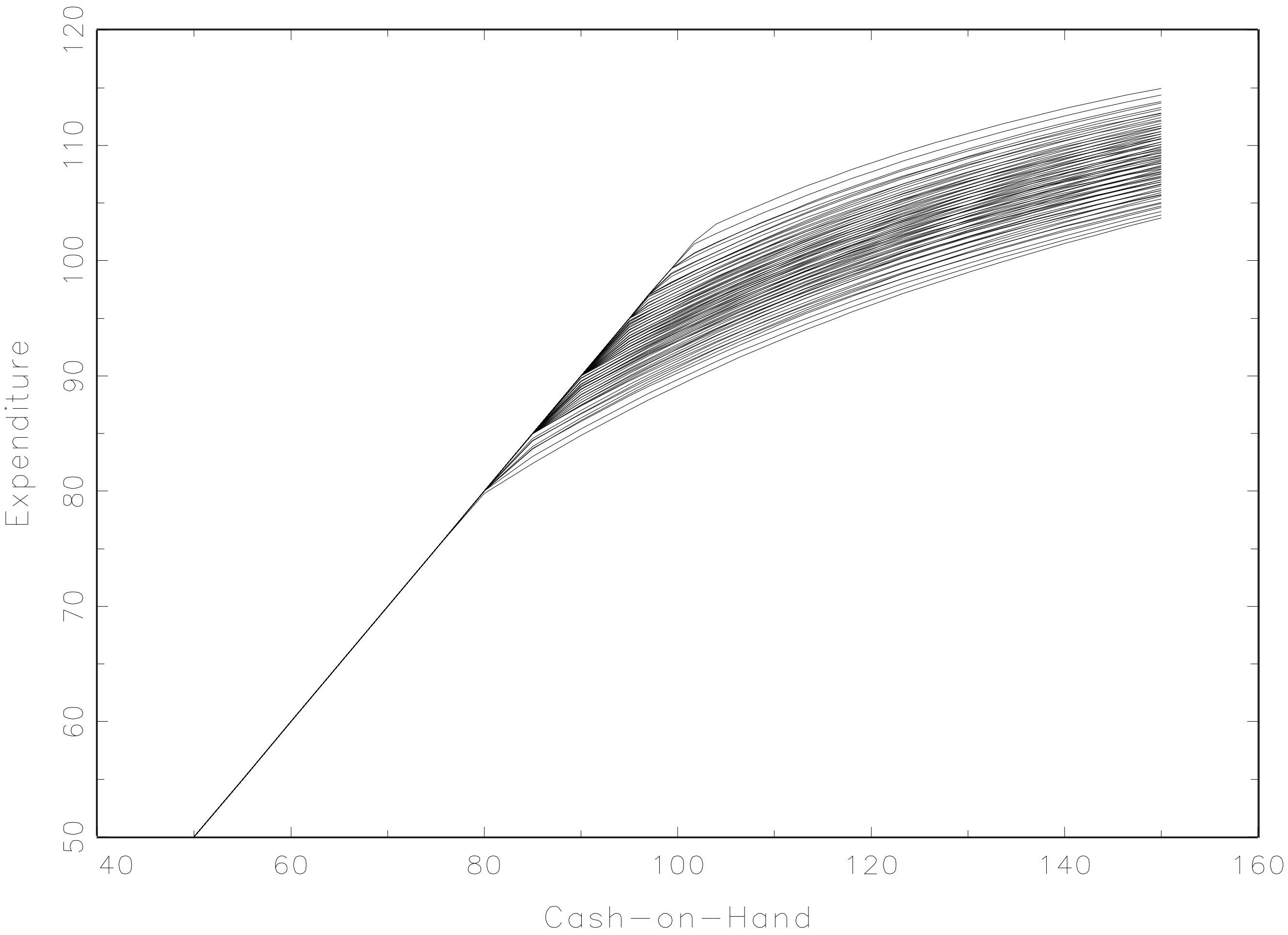

Figure 2: Percentage of Private Debt in GDP

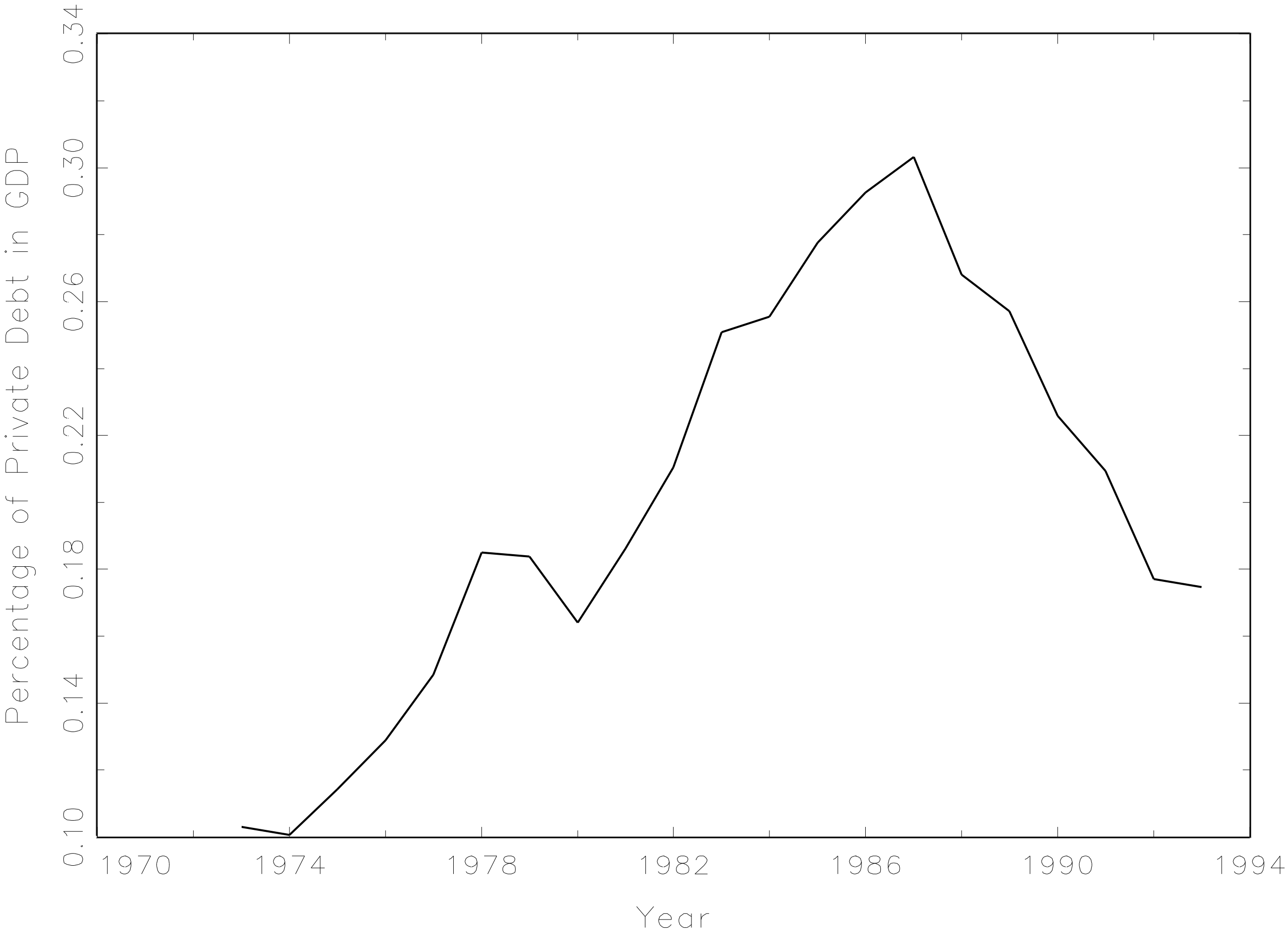

Figure 3: Estimated Credit Ceilings and Mean Income

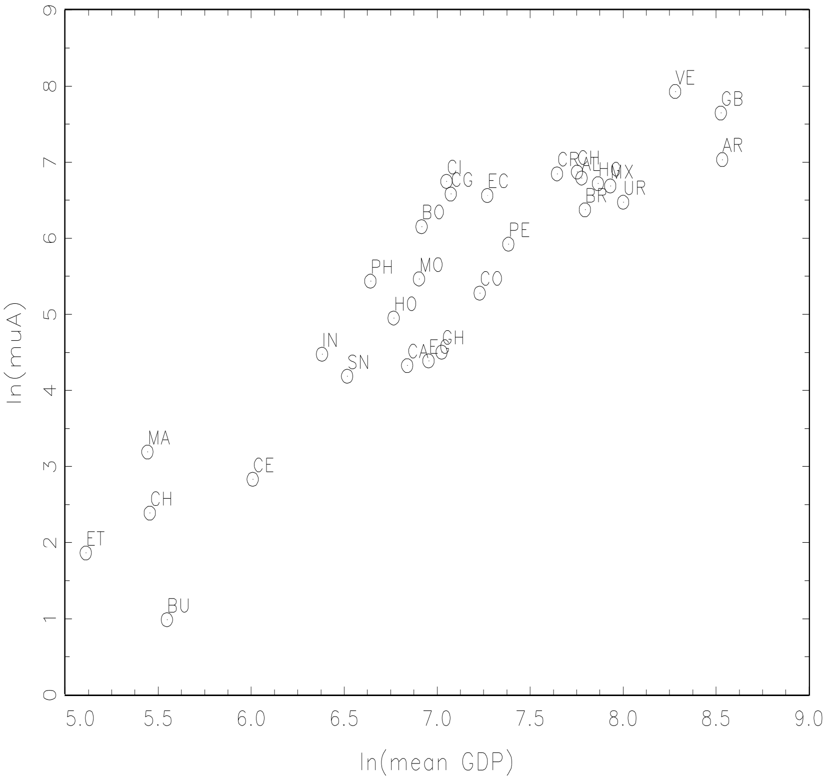

Figure 4: Brazil, Dynamic of Debt and Cash-on-Hand

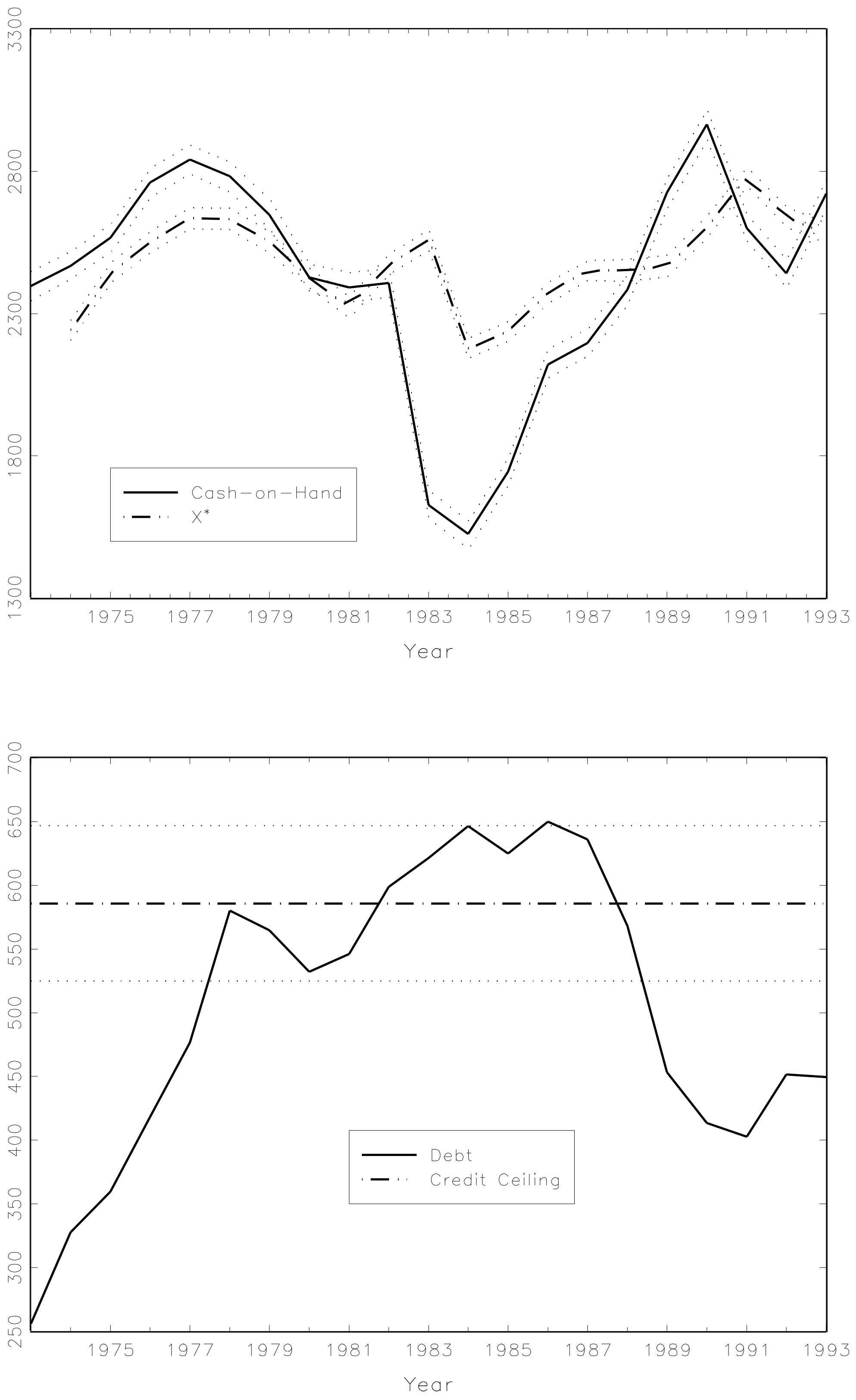

Figure 5: Proportion of countries with binding liquidity constraints