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by

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C E P R E M A P

Monopolistic price setting and equilibrium have had quite a long history in Economics (Chamberlain [12], Robinson [33], Triffin [35]) and still most contributions to General Equilibrium Theory continue to view the firm as a price taker. Recently Arrow [1], noticing that "there is no one left over whose job is to make a decision on price" advocated for a more realistic approach to price determination, by firms behaving monopolistically and stressed particularly the relation between monopolistic and out-of-equilibrium behavior. Monopolistic price setting was incorporated for the first time in a General Equilibrium model in a brilliant paper by Negishi [29]. A number of other studies followed (1).

We shall retain in this study a basic feature of Negishi's paper : The perceived demand curve, a concept introduced in Bushaw, Clower [11]. The perceived demand curve gives the maximum quantity of monopolized good that the monopolist thinks to be able to sell as a function of his price, given his market observations. Such a subjective perception is clearly much more realistic than the assumption that the monopolist knows the "true" demand curve facing him (Negishi [30]). So, each time the monopolist has to make a price decision, he reestimates his perceived demand curve in function of what he observes (notably his maximal possible sales) and then chooses the price of the goods he controls in function of this perceived demand curve and his technological possibilities.

(*) This paper is based on some chapters of an unpublished Ph.D. Thesis [6] for which I had the precious help of Gerard Debreu and Bent Hansen. I also wish to thank, for discussion and comments M. Allingham, G. Laroque and an anonymous referree.

(1) Negishi [30] [31] [32]. See also [2] [3] [4] [19] [22] [23] [25] [28].

However, the models presented until now were generally tatonnement models in prices so that sales could actually be observed only at the general monopolistic equilibrium point. As the perceived demand curve theory is based on the observation of actual sales possibilities, this makes the whole process of price setting by individual firms somewhat unrealistic, since based on completely fictitious observations (and actually the process generally described looks very much like the usual "auctioneer" process, with "deviant" monopolistic firms). Also it is not clear either where the whole family of perceived demand curves comes from, with so few observations.

So what is clearly most needed here is a model of non-tatonnement in prices where transactions can actually happen outside equilibrium. For that we shall use an assumption which is common in Keynesian or disequilibrium theories : quantities adjust infinitely faster than prices. So the dynamics of the economic system can be described as follows : Assume that all firms have fixed their prices ; quantity movements occur, with eventual multiplier effects, then quantities stabilize at what we will call a K-equilibrium, and transactions can actually take place. Firms then observe some price and quantity variables, reestimate their perceived demand curves, change their prices, and so on ...

Monopolistic equilibrium is attained when no monopolist wants to change his price on the basis of what he observes. This is the process we shall describe in the next sections. After defining the Economy (section I), we will study first quantity adjustments for fixed prices (section II), then monopolistic price setting (section III), and prove the existence of a monopolistic equilibrium in our framework (section IV). A number of simple extensions will be considered in section V.

I. - THE ECONOMY

1. MARKETS AND AGENTS.

The Economy described will be a pure flow monetary economy "A la Clower" [15]. There will be ℓ markets on which money is exchanged against each of the ℓ non-monetary goods ($h \in \{1 \ldots \ell\} = H$) (2). An important feature we should remember is that demands on these ℓ markets are expressed separately (as they are indeed in reality). p_h is the monetary price of good h. The price of money is one.

The agents in the Economy, will be consumers (indexed by $i \in I$) and monopolistic price setting firms (indexed by $j \in J$).

Consumer i has an initial endowment (ω_i , $\bar{M}_i$) of non monetary goods and money respectively $\omega_i \in R^\ell_+$ $\bar{M}_i \in R_+$. His utility function is :

$$U_i[x_i , M_i] = U_i[\omega_i + z_i , M_i] \qquad (3)$$

Where $x_i \in R^\ell_+$ is his final consumption vector, $z_i \in R^\ell$ his net trade vector, $M_i \geq 0$ his final holding of money. We shall assume that goods can be partitioned into a set of demanded goods (D_i) and supplied goods (S_i). So if we call z_{ih} the net demand of good h :

$$z_{ih} \geq 0 \quad h \in D_i \qquad\qquad z_{ih} \leq 0 \quad h \in S_i$$

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(2) As noted by Clower [15][16] (see also [9]), the notion of demand for one good is unambiguous only in an economy with a unique medium of exchange.

(3) Money will be taken here to be a consumable commodity money. Storable fiat money can be introduced in a disequilibrium analysis [6] [8] but would have made notations too heavy here.

We shall make the following assumptions on utilities and endowments :

. U_i is continuous, concave and increasing in its arguments

. $\bar{M}_i > 0$

The producer j combines different inputs ($h \in D_j$) to produce outputs ($h \in S_j$). A production plan is a vector $y_j \in Y_j \subset R^\ell$, where Y_j is j's production set (4) (We shall also sometimes work with j's net demand $z_j = - y_j$). With the usual sign convention :

$$y_{jh} \geq 0 \qquad h \in S_j \qquad , \qquad y_{jh} \leq 0 \qquad h \in D_j$$

We make on production sets the usual assumptions (Debreu [17]) :

. Y_j closed, convex

. $0 \in Y_j$

. $Y \cap (-Y) = \{0\}$ with $Y = \sum_j Y_j$

2. PRICE SETTING.

As we said, we want to emphasize the price setting behavior of agents internal to the economy. We shall now assume that firms control all prices of non-monetary goods. These goods will be distinguished by their physical characteristics and the firm which controls their price, so that each firm is, at least formally, a true monopolist (or monopsonist) on its markets. Thus each non-monetary good will have its price controlled by one (and only one) of the monopolists. If we call H_j the set of goods controlled by firm j , we will have :

$$\bigcup_{j \in J} H_j = H \qquad H_j \cap H_{j'} = \{\emptyset\} \qquad j \neq j'$$

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(4) Production is thus "instantaneous". A more realistic assumption would be to have production take one, or more, periods (cf. Grandmont-Laroque [19], Iwaï [23]). This would have made the non-tatonnement too heavy.

We shall also denote by :

$p_j \in R_+^{H_j}$ the vector of prices controlled by j

$y_{H_j} \in R^{H_j}$ the part of the production vector of firm j related to the goods it controls.

3. THE TIME-STRUCTURE OF TRADING.

Trading in our model will actually take place in a sequence of periods, indexed by t. Price decisions are made by all firms at the beginning of each period, i.e., each firm sets the prices $p_j(t)$ which will prevail during period t. Quantity adjustments, which we assume infinitely fast, take place, and a quantity equilibrium (K-equilibrium) establishes. Agents express demands $\tilde{z}_{ih}(t)$, $\tilde{y}_{jh}(t)$. Then trading occurs on each market yielding realized transactions $\bar{z}_{ih}(t)$, $\bar{y}_{jh}(t)$. During this trading process, the agents perceive constraints on their sales (or purchases) $\bar{\bar{z}}_{ih}(t)$, $\bar{\bar{y}}_{ih}(t)$. These constraints, which are quantity signals, are the main informational link between successive periods. Together with the regular price signals, they will determine prices chosen in the next period $p_j(t+1)$. We turn now to the first step of this process.

II. - QUANTITY ADJUSTMENTS AND EQUILIBRIUM WITH FIXED PRICES.

Assume prices $p_h(t)$ have been quoted for all goods at the beginning of period t (From now on this index t will be omitted, as everything refers to the same trading period). We shall see how the different quantities are determined in the quantity equilibrium (K-equilibrium) which will establish. This description will be extremely sketchy as it has been developed at length, in tatonnement and non-

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tatonnement versions, in a previous paper (Benassy [8]), to which the reader is referred (5).

1. CONSTRAINTS IN DISEQUILIBRIUM AND EFFECTIVE DEMANDS.

As we said, we allow in our model trades to occur at non Walrasian prices. Thus some traders will not be able to fulfill their demands and feel constrained on their trades. More precisely, each trader will perceive an upper bound on the amount of trade he can realize on this particular market. Beyond this bound his demand (or supply) is not satisfied.

We shall call $\bar{\bar{z}}_{ih}$ and $\bar{\bar{y}}_{jh}$ respectively these perceived constraints. As an example, trades perceived as possible will have the form : $z_{ih} \leqslant \bar{\bar{z}}_{ih}$ for a good demanded by a consumer ($h \in D_i$).

Now, rational consumers (or firms), if they cannot transact what they want on some markets, will modify their demands and supplies on all other markets, taking account of the constraints they perceive, as was indicated by Clower [14]. These "constrained" demand functions will be called effective demands.

So we shall call effective demand of a consumer i on a market h (which we shall denote $\tilde{z}_{ih}$) the demand he will formulate by maximizing his utility subject to the usual budget and positivity constraints, <u>and</u> the quantity constraints he perceives on the <u>other</u> markets. So $\tilde{z}_{ih}$ will be the h-th component of the vector solution of the following program :

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(5) The concepts of effective demand and quantity adjustment used here were proposed initially by Clower [14] and Leijonhufvud [26]. A different tatonnement for rigid prices can be found in Drèze [18] Grandmont-Laroque [19].

Maximize $U_i\ [\omega_i + z_i\ ;\ M_i]$ s.t.

$$\begin{cases} pz_i + M_i \leqslant \bar{M}_i + w_i & (6) \\ \omega_i + z_i \geqslant 0 \qquad M_i \geqslant 0 & \\ z_{ih'} \leqslant \bar{\bar{z}}_{ih'} \qquad h' \in D_i \qquad h' \neq h & \\ z_{ih'} \geqslant \bar{\bar{z}}_{ih'} \qquad h' \in S_i \qquad h' \neq h & \end{cases}$$

This last set, the perceived choice set of the individual, we shall note $\gamma_{ih}\ [p,\ \bar{\bar{z}}_i\ ,\ w_i]$.

In the same way, firm j's effective supply of good h, $\tilde{y}_{jh}$ will be found by maximizing profit subject to technological possibilities and the quantity constraints perceived on other markets : $\tilde{y}_{jh}$ will be the h-th component of the optimizing vector of :

Maximize $p.y_j$ subject to

$$\begin{cases} y_j \in Y_j & \\ y_{jh'} \geqslant \bar{\bar{y}}_{jh'} \qquad h' \in D_j \qquad h' \neq h & \\ y_{jh'} \leqslant \bar{\bar{y}}_{jh'} \qquad h' \in S_j \qquad h' \neq h & \end{cases}$$

This last set, the perceived choice set of the firm, we call $Y_{jh}(\bar{\bar{y}}_j)$ (or $Y_{jh}(\bar{\bar{z}}_j)$ with evidently $\bar{\bar{z}}_j = -\bar{\bar{y}}_j$).

So we see that effective demand functions differ essentially from the neoclassical ones by the fact that traders do not respond only to prices, but also to the quantity constraints they may experience on the different markets.

2. TRANSACTIONS, RATIONING AND CONSTRAINTS PERCEPTION.

Consider now a particular market h : once demands have been expressed, transactions will take place in a decentralized way. Demand and supply usually do not sum up to zero :

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(6) With $w_i = \sum_{j\in J} \theta_{ij}\ \pi_j$ where π_j is firm j's profits and θ_{ij} i's share in j's profits (See Debreu [17]).

$$\tilde{z}_h = \sum_{i=1}^{n} \tilde{z}_{ih} \neq 0 \qquad (7)$$

So a rationing scheme is necessary to obtain actual transactions $\bar{z}_{ih}$. We shall assume :

$$\bar{z}_{ih} = F_{ih}\,[\tilde{z}_{1h}, \ldots, \tilde{z}_{nh}]$$

with $\sum_{i=1}^{n} F_{ih}\,[\tilde{z}_{1h}, \ldots, \tilde{z}_{nh}] \equiv 0.$

The rationing functions have the following properties (8) :

. Voluntary exchange :

- $|\bar{z}_{ih}| \leq |\tilde{z}_{ih}|$ and $\bar{z}_{ih}.\tilde{z}_{ih} \geq 0$

. Agents on the "short side" realize their demands :

- $\tilde{z}_h.\tilde{z}_{ih} \leq 0 \implies \bar{z}_{ih} = \tilde{z}_{ih}$

. The F_{ih} are continuous.

During this trading process the agents perceive limits on their trading possibilities on market h, $\bar{\bar{z}}_{ih}$. We assume :

$$\bar{\bar{z}}_{ih} = G_{ih}\,[\tilde{z}_{1h}, \ldots, \tilde{z}_{nh}]$$

This constraint is equal to the transaction realized if the agent is contrained, greater if he is unconstrained, and varies continuously with demands :

- $|\bar{z}_{ih}| < |\tilde{z}_{ih}| \implies \bar{\bar{z}}_{ih} = \bar{z}_{ih}$
- $\bar{z}_{ih} = \tilde{z}_{ih} \implies (\bar{\bar{z}}_{ih} - \bar{z}_{ih}).\tilde{z}_{ih} \geq 0$
- $\tilde{z}_{ih}.\tilde{z}_h < 0 \implies (\bar{\bar{z}}_{ih} - \bar{z}_{ih}).\tilde{z}_{ih} > 0$
- The G_{ih} are continuous in their arguments.

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(7) We take here a unique index i, running from 1 to n, for individuals and firms.

(8) These have been emphasized in Clower [13][14], Barro-Grossman [5], Grossman [20], Howitt [21].

We can remark that, even if an agent could realize his desired transaction on a market, he will never feel totally unconstrained on that market ($\bar{\bar{z}}_{ih}$ infinite). This is quite natural since if he were to increase his demand, then at some point this increased demand would put him on the "long" side. He is thus effectively constrained on that market, even though he can carry all the transactions he presently wishes.

3. K-EQUILIBRIUM.

It is easy to see that, if one just takes an arbitrary set of effective demands $\tilde{z}_{ih}$, perceived constraints $\bar{\bar{z}}_{ih}$, transactions $\bar{z}_{ih}$, these will be in general inconsistant with respect to the relations seen above. So there will be a process of quantity adjustments, which resembles very much to the traditional Keynesian multiplier process, in which people revise their expectations on the constraints they will perceive, and modify accordingly their effective demands. This recursive adjustment process can be pictured simply as follows (9) :

Assume initially consumers and firms have expressed effective demands and supplies $\tilde{z}_{ih}$, $\tilde{y}_{jh}$. We deduce from these realized transactions and perceived constraints (Here again index i applies to firms as well as households) :

$$\bar{z}_{ih} = F_{ih} [\tilde{z}_{1h} ,\ldots, \tilde{z}_{nh}]$$

$$\bar{\bar{z}}_{ih} = G_{ih} [\tilde{z}_{1h}, \ldots, \tilde{z}_{nh}]$$

Effective demands in the next "round" $\tilde{\tilde{z}}_{ih}$, will be given by the known programs :

Maximize $U_i [\omega_i + z_i , M_i]$ in $\gamma_{ih} [p, \bar{\bar{z}}_i , w_i]$.

Similarly new effective supplies of firms $\tilde{\tilde{y}}_{jh}$ will be given by :

Maximize py_j in $Y_{jh} [\bar{\bar{y}}_j]$.

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(9) Evidently this is here a fictitious process of tatonnement in quantities. For a (more complicated) description of a non-tatonnement process in "real time", see Benassy [8, appendix].

An equilibrium for fixed prices, which we will call a K-equilibrium, is a fixed point of the above recursive process, i.e., a set of self-reproducing effective demands.

We can summarize here the main characteristics of a K-equilibrium : Individuals and firms express on each market effective demands and supplies $\tilde{z}_{ih}$, $\tilde{y}_{jh}$. They realize transactions $\bar{z}_{ih}$, $\bar{y}_{jh}$ which may be smaller than their demands if they are on the "long" side. (So there can be unemployment, rationing, or any form of market imbalance at a K-equilibrium).

Correspondingly they perceive limits on their trading possibilities on each market, $\bar{\bar{z}}_{ih}$, $\bar{\bar{y}}_{jh}$, which are equal to the transactions they realize (if they are constrained) or greater than these transactions (if they are not constrained).

A K-equilibrium is a perfectly observable state of the economy, where transactions can actually take place. These transactions will be consistent both at the global and individual level :

- At the economy's level, since we imposed on each market the basic accounting identity :

$$\sum_{i=1}^{n} \bar{z}_{ih} \equiv 0$$

- At the individual level, transactions realized by each agent are rational, since they are the best possible, given all the constraints he perceives, i.e.,

$$\bar{z}_i \text{ maximizes } U_i[\omega_i + z_i , M_i] \qquad \text{s.t.}$$

$$\begin{cases} pz_i + M_i \leq \bar{M}_i + w_i \\ \omega_i + z_i \geq 0 \qquad M_i \geq 0 \\ z_{ih} \leq \bar{\bar{z}}_{ih} \qquad h \in D_i \\ z_{ih} \geq \bar{\bar{z}}_{ih} \qquad h \in S_i \end{cases}$$

and similarly

$$\bar{y}_j \text{ maximizes } py_j \qquad \text{s.t.}$$

$$\begin{cases} y_j \in Y_j \\ y_{jh} \leq \bar{\bar{y}}_{jh} \qquad h \in S_j \\ y_{jh} \geq \bar{\bar{y}}_{jh} \qquad h \in D_j \end{cases}$$

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Also we should remark, as did Iwai [22][23], that usually a firm will not satisfy the demand (or supply) of the goods it controls, contrarily to the usual view of the monopolist "clearing the market", so that there may well be unemployment or rationing on the markets controlled by the monopolist. As we shall see, clearing the market is just part of the price-optimizing behavior of the monopolist.

III. - MONOPOLISTIC PRICE SETTING AND EQUILIBRIUM.

Having studied what happens for each set of prices quoted, we shall now determine how the monopolists set their prices at the beginning of each period. We start by studying a main element in their decision, the perceived demand curve.

1. PERCEIVED DEMAND CURVES OF THE MONOPOLISTS.

While the perceived constraints on the goods he does not control are given for the monopolist, he can modify them for the controlled goods by changing his prices. The relation between prices and perceived constraints will be particularly important in the monopolist's price decisions. This is the well-known "perceived demand curve" (Bushaw, Clower [11]) which gives the maximum quantity of controlled goods the monopolist expects to be able to sell (or to buy if it is a production input) as a function of the prices he will set.

This curve will not be a given one (as would be if the monopolist knew the "true" demand curve), but rather it will be adjusted by the monopolist as a function of his information. We shall assume to start that the monopolist takes into account only the information gathered in the previous period (this is generalized in the last section). This information consists of :

- $p_j(t-1)$ and $\bar{\bar{y}}_{H_j}(t-1)$, i.e. the price set in the previous period by the monopolist for the goods he controls, as well as the constraints he has perceived on the corresponding markets. This is evidently the minimal information the monopolist will have.

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. $I_j(t-1)$ which represents the set of all other information variables known to the monopolist at the end of period t-1 and relevant to him (this may include some competitiors' prices for example).

Thus, if we note $\bar{\bar{y}}^e_{H_j}(t)$ the maximal sales (or purchases) which the monopolist <u>expects</u> to be able to make in period t on the markets he controls, the perceived demand curve will be written :

$$\bar{\bar{y}}^e_{H_j}(t) = \bar{\bar{Y}}_j \, [p_j \mid \bar{\bar{y}}_{H_j}(t-1), \, p_j(t-1), \, I_j(t-1)]$$

A vector of dimension H_j with components $\bar{\bar{Y}}_{jh}$ $(h \in H_j)$.

We shall ask as a logical requirement that the perceived demand curve is consistent with the data of the previous period, i.e., that it goes through the just observed point :

$$\bar{\bar{y}}_{H_j}(t-1) \equiv \bar{\bar{Y}}_j \, [p_j(t-1) \mid \bar{\bar{y}}_{H_j}(t-1), \, p_j(t-1), \, I_j(t-1)] \qquad (10).$$

<u>Remark</u>. As the monopolist is the only seller (or buyer) on the markets he controls, we can assume that he knows at least the total demand addressed to him by the other agents, and thus we can take :

$$\bar{\bar{y}}_{jh}(t) = \sum_{i \in I} \tilde{z}_{ih}(t) - \sum_{\substack{j' \in J \\ j' \neq j}} \tilde{y}_{jh}(t) \qquad h \in H_j$$

This was the formulation taken by Negishi [29].

Though, as presented here, not all perceived demand curves described with that generality are realistic, since the perceived demand for a good controlled by a monopolist can depend in any possible fashion upon the prices of the other goods he controls. The following specifications are particularly realistic (Negishi [30]) :

. In the "normal" case where the monopolist controls rather different products, he should perceive no direct dependence between them :

$$\frac{\partial \bar{\bar{Y}}_{jh}}{\partial p_{h'}} = 0 \qquad h, h' \in H_j \qquad h \neq h'.$$

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(11) Cf. Bushaw-Clower [11] Negishi [29].

In this case the components of the perceived demand curve can be written :

$$\bar{\bar{y}}_{jh}\,[p_h \mid \bar{\bar{y}}_{H_j}(t-1),\ p_j(t-1),\ I_j(t-1)].$$

. However the monopolist may practice product differentiation, and will in this case perceive the corresponding goods as substitutes :

$$\frac{\partial \bar{\bar{y}}_{jh}}{\partial p_{h'}} \geq 0 \quad \text{for } h, h' \in S_j \quad \text{or} \quad h, h' \in D_j \,,$$

the other cross-derivatives being zero.

We shall only consider these two cases in all what follows.

2. <u>MONOPOLISTIC PRICE SETTING</u>.

Once the perceived demand curve is known, the problem of the monopolist is simple : he will choose his price vector $p_j(t)$ so as to maximize his profit subject to :

- . his production possibilities,
- . his perceived demand curve,
- . the prices and quantity constraints he expects on the markets he does not control ; we shall take these to be equal to last period's observed ones for simplicity.

So the program giving the optimal price will be written :

$$\text{Maximize} \quad \sum_{h \in H_j} p_h\, y_{jh} + \sum_{h \notin H_j} p_h(t-1)\, y_{jh} \quad \text{subject to :}$$

$$\begin{cases} y_j \in Y_j \\ y_{jh} \leq \bar{\bar{y}}_{jh}[p_j \mid \bar{\bar{y}}_{H_j}(t-1),\ p_j(t-1),\ I_j(t-1)] & h \in S_j \quad h \in H_j \\ y_{jh} \geq \bar{\bar{y}}_{jh}[p_j \mid \bar{\bar{y}}_{H_j}(t-1),\ p_j(t-1),\ I_j(t-1)] & h \in D_j \quad h \in H_j \\ y_{jh} \leq \bar{\bar{y}}_{jh}(t-1) & h \in S_j \quad h \notin H_j \\ y_{jh} \geq \bar{\bar{y}}_{jh}(t-1) & h \in D_j \quad h \notin H_j \end{cases}$$

It yields the optimal monopolistic price vector together with the expected production plan (this last one is however irrelevant since expectations will not in general be fulfilled). We shall assume that the solution (in p_j) of this program is unique. The result will be an optimal monopolistic price function :

$$p_j(t) = p_j^* [p_h(t-1), \bar{\bar{y}}_{jh}(t-1), I_j(t-1)].$$

The goods whose prices and quantity limits p_h and $\bar{\bar{y}}_{jh}$ appear as arguments are not all goods, but only the ones the monopolist trades (12).

We can already make a simple but important remark about the prices the monopolist is likely to choose, for a given set of perceived demand curves :

The monopolist will always choose his prices so as to be on the "long" side of all the markets he controls, so that the perceived constraints given by his perceived demand curves will be <u>actually binding</u> : Indeed, if he were on the short side of one market he controls, he could increase the price of the product (or reduce it if it was the price of a factor of production), and still be able to carry the same production and sales plan in quantities, thus increasing his profit. So we see here that the fact that the monopolist "clears" the market (i.e. is on the long side), is a profit maximization condition, and should not be an a priori hypothesis.

3. MONOPOLISTIC EQUILIBRIUM.

The dynamics of our economy are now quite simple to describe : let us start from any period t : prices $p_j(t)$ have been quoted by the monopolists. A K-equilibrium establishes where demands $\tilde{z}_{ih}(t)$, $\tilde{y}_{jh}(t)$ are expressed, transactions $\bar{z}_{ih}(t)$, $\bar{y}_{jh}(t)$ realized, constraints $\bar{\bar{z}}_{ih}(t)$, $\bar{\bar{y}}_{jh}(t)$ perceived. On the basis of the information gathered in t, monopolists will determine a new set of prices in the next period $p_j(t+1)$.

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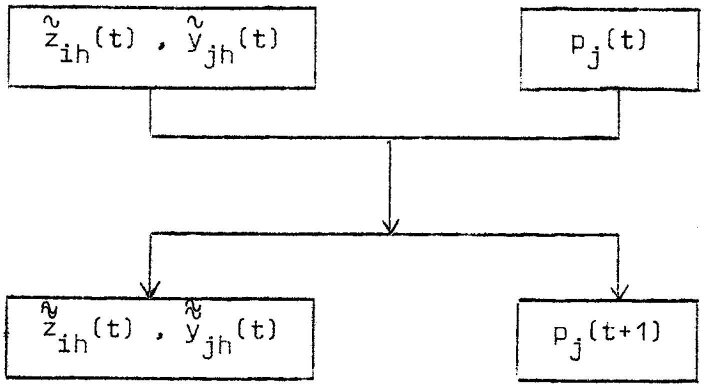
(12) So, for example, the prices of his competitors will not be an argument of p_j^* , unless they appear in I_j .

In this dynamic trading framework, the most natural definition of a monopolistic equilibrium is that of a self-reproducing state in time, i.e. a K-equilibrium such that the perceived constraints and other information he receives would lead each monopolist to choose in the next period the same prices as in the current one. It is easy to see that a monopolistic equilibrium will be a fixed point of the following mapping :



with :

- $\tilde{\tilde{z}}_{ih}(t)$ maximizes $U_i[\omega_i + Z_i , M_i]$ over γ_{ih} $[p(t), \bar{\bar{z}}_i(t), w_i(t)]$
- $\tilde{\tilde{y}}_{jh}(t)$ maximizes $p(t)\ y_j$ over Y_{jh} $[\bar{\bar{y}}_j(t)]$
- $p_j(t+1) = p_j^* [p_h(t), \bar{\bar{y}}_{jh}(t), I_j(t)]$

This mapping can be thought of as consisting of two sub-mappings :

- The mapping : $\tilde{z}_{ih}(t) \rightarrow \tilde{\tilde{z}}_{ih}(t)$ $\tilde{y}_{jh}(t) \rightarrow \tilde{\tilde{y}}_{jh}(t)$ represents the intra-period adjustment process in quantities.
- The mapping : $p_j(t) \rightarrow p_j(t+1)$ represents the price revision from one period to the next which occurs once a K-equilibrium has been established.

As the reader can check, monopolistic equilibrium as defined above will have the same characteristics as the ones generally considered in the literature (e.g. Negishi [29]). In particular, at equilibrium, the monopolist will be really on the "long" side for all markets he controls (and not only with respect to his perceived demand curves), so that he will actually "clear" these markets.

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IV. - THE EXISTENCE OF A MONOPOLISTIC EQUILIBRIUM.

In order to show that a monopolistic equilibrium exists, we have to show that the above mapping in an upper semi-continuous (u.s.c.) mapping with convex values from a compact convex set into itself. Here a difficulty might arise from the possible unboundedness of prices and effective demands. Actually our assumptions will ensure that optimal prices are bounded. As for effective demands, our procedure, taken from Debreu's Theory of Value([17] especially section 5.7) will be the following : first, we shall show that effective demands are bounded at equilibrium, and thus belong to the interior of a suitably chosen compact convex set K. Then, when we shall study the mappings, it will be implicitly understood that the choice sets (Production, consumption sets) are reduced to their intersection with K. The desired properties will be proved for these modified mappings. The last step would be to prove that, at equilibrium, the restriction to K does not modify the agents'actions. The proof would be completely similar to Debreu's one, and we shall thus omit it.

1. ASSUMPTIONS.

In addition to the classical assumptions on the agents, listed in section I.1, we will have to make a few more assumptions on the price making behavior of the monopolists.

The first one has already been seen :

A1 : The optimal price charged by the monopolist p_j^* is unique.

Also we would like to express that a monopolist has not complete monopolistic (or monopsonistic) power over the markets he controls, i.e. that he has not interest to charge a too high price on the products he sells (or to fix a too low price on the factors he buys). More specifically, we shall assume

A2 : For every good there exists $\bar{p}_h > 0$ such that, for all observable $p_j(t)$, $I_j(t)$, $\bar{\bar{y}}_{H_j}(t)$, we have :

$$p_j^* [p_h(t), \bar{\bar{y}}_{jh}(t), I_j(t)] \leq \bar{p}_h \qquad h \in H_j \cap S_j$$
$$p_j^* [p_h(t), \bar{\bar{y}}_{jh}(t), I_j(t)] \geq \bar{p}_h \qquad h \in H_j \cap D_j$$

We could have obtained this result by assuming that demand (resp. supply) goes to zero for $p_h > \bar{p}_h$ (resp. $p_h < \bar{p}_h$). We prefer however the above formulation which allows for more general demand and supply functions. Notice that this assumption is quite realistic because we assumed away here any monetary creation. Finally we shall make quite natural continuity assumptions :

<u>A3</u> : . The inverse demand curve

$$p_j = \mathcal{P}_j [\bar{\bar{y}}_{H_j} \mid p_j(t), \bar{\bar{y}}_{H_j}(t), I_j(t)]$$

is continuous in its arguments.

. I_j is continuous in prices and effective demands.

We can now proceed to the proof of boundedness, upper semi-continuity and convexity of the above correspondence. The time index will be dropped each time there is no risk of ambiguity.

2. <u>BOUNDEDNESS</u>.

By our assumption of absence of complete monopoly power, we have :

$$p_h^* \leq \bar{p}_h \qquad h \in H_j \cap S_j .$$

This will make the receipts of the firm finite, and thus the prices of inputs p_h^* $(h \in H_j \cap D_j)$ must also be bounded.

As for effective demands the argument goes as follows :

. Transactions and effective demands of an agent are equal for the goods he does not control :

$$\tilde{z}_{ih} = \bar{z}_{ih} \quad \forall\, i \in I \quad \forall\, h$$
$$\tilde{y}_{jh} = \bar{y}_{jh} \quad \forall\, j \quad \forall\, h \notin H_j$$

As our assumptions ensure boundedness of transactions, these effective demands will be bounded also.

- $\tilde{y}_{jh}$ $h \in H_j \cap S_j$, i.e. monopolists' supplies of the goods they control will be bounded, because of the boundedness of available inputs.
- $\tilde{y}_{jh}$ $h \in H_j \cap D_j$, i.e. monopolists' demands of goods they control will be bounded because for these goods $p_h^* \geq \bar{p}_h > 0$, and thus with limited receipts a too high demand would make profits negative.

3. UPPER SEMI-CONTINUITY AND CONVEXITY.

a) $\tilde{\tilde{y}}_{jh}$

The choice set $Y_{jh}[\bar{\bar{y}}_j]$ is convex and varies continuously with effective demands. As the maximand function py_j is concave and continuous $\tilde{\tilde{y}}_{jh}$ will be a convex and u.s.c. mapping.

b) $\tilde{\tilde{z}}_{ih}$

The choice set $\gamma_{ih}[p, \bar{\bar{z}}_i , w_i]$ is convex and varies continuously with its arguments, and thus effective demands and prices. The continuity proof is due to Drèze [18], and uses the fact that $\bar{M}_i > 0$ (strictly positive initial endowment of money). The maximand function U_i is continuous and concave, and thus $\tilde{\tilde{z}}_{ih}$ is a convex and u.s.c. mapping.

c) $p_j(t+1)$

The proof here will be a little more indirect. We shall use the assumption that the price solution to the monopolist's optimization program is unique, so that we need only to prove upper semicontinuity (then equivalent to continuity).

Remember the original program is :

$$\text{Maximize} \sum_{h \in H_j} p_h \, y_{jh} + \sum_{h \notin H_j} p_h(t) \, y_{jh} \qquad \text{subject to :}$$

.../...

$$\begin{cases} y_j \in Y_j \\ y_{jh} \leq \bar{\bar{y}}_{jh}[p_j \mid \bar{\bar{y}}_{H_j}(t),\ p_j(t),\ I_j(t)] & h \in S_j \quad h \in H_j \\ y_{jh} \geq \bar{\bar{y}}_{jh}[p_j \mid \bar{\bar{y}}_{H_j}(t),\ p_j(t),\ I_j(t)] & h \in D_j \quad h \in H_j \\ y_{jh} \leq \bar{\bar{y}}_{jh}(t) & h \in S_j \quad h \notin H_j \\ y_{jh} \geq \bar{\bar{y}}_{jh}(t) & h \in D_j \quad h \notin H_j \end{cases}$$

The solution to this program will be the price and the expected production of controlled goods in period t+1 $(p_j^*\ ,\ y_{H_j}^*)$.

But, as we noted earlier, prices chosen will be always such that the perceived demand curves constraints will be binding ; i.e., if we use the inverse demand curve, we will have :

$$p_j^*(t+1) = \mathcal{P}_j\ [y_{H_j}^*(t+1) \mid p_j(t),\ \bar{\bar{y}}_{H_j}(t),\ I_j(t)].$$

Using again the same property, we see that $y_{H_j}^*(t+1)$ itself will be solution of :

$$\text{Maximize } \mathcal{P}_j\ [y_{H_j} \mid p_j(t),\ \bar{\bar{y}}_{H_j}(t),\ I_j(t)]\ y_{H_j} + \sum_{h \notin H_j} p_h(t)\ y_{jh}$$

subject to :

$$\begin{cases} y_j \in Y_j \\ y_{jh} \leq \bar{\bar{y}}_{jh}(t) & h \in S_j \quad h \notin H_j \\ y_{jh} \geq \bar{\bar{y}}_{jh}(t) & h \in D_j \quad h \notin H_j \end{cases}$$

The result will be a mapping :

$$y_{H_j}^*(t+1) = y_{H_j}^*[p_h(t),\ \bar{\bar{y}}_{jh}(t),\ I_j(t)]$$

which is clearly u.h.c. in its arguments, since the maximand function is continuous in its arguments, and the set over which it is maximized also.

Since $y^*_{H_j}$ is u.h.c.

$$p_j(t+1) = \mathcal{P}_j\ [y^*_{H_j}(t+1) \mid p_j(t),\ \bar{\bar{y}}_{H_j}(t),\ I_j(t)]$$

is also u.h.c. (and thus continuous).

Q.E.D.

We now have the following

Theorem. Under the assumptions given in I.1, and IV.1, a monopolistic equilibrium exists.

V. - EXTENSIONS.

We shall give here two extensions of the model which could have been trivially included in it, except for the increased notations. These are respectively :

- The possibility of having price setting households (for example workers setting their wages) (13).
- The inclusion of many period's observations for the determination of the perceived demand curves.

1. Assume thus there are some price setting households. Let H_i be the set of goods controlled by i, p_i the price vector and z_{H_i} the excess demand vector corresponding to these goods. The perceived demand curve will be written :

$$\bar{\bar{z}}^e_{H_i}(t) = \bar{\bar{Z}}_i\ [p_i \mid \bar{\bar{z}}_{H_i}(t-1),\ p_i(t-1),\ I_i(t-1)]$$

and the program giving the optimal monopolistic price in t :

Maximize $U_i\ [\omega_i + z_i\ ,\ M_i]$ subject to

.../...

---

(13) This possibility is found in Grandmont-Laroque [19], Negishi [31][32].

$$
\begin{cases}
\sum_{h \in H_i} p_h z_{ih} + \sum_{h \notin H_i} p_h(t-1) z_{ih} + M_i \leq \bar{M}_i \\
z_{ih} \leq \bar{\bar{Z}}_{ih} [p_i \mid \bar{\bar{z}}_{H_i}(t-1), p_i(t-1), I_i(t-1)] \quad h \in D_i \quad h \in H_i \\
z_{ih} \geq \bar{\bar{Z}}_{ih} [p_i \mid \bar{\bar{z}}_{H_i}(t-1), p_i(t-1), I_i(t-1)] \quad h \in S_i \quad h \in H_i \\
z_{ih} \leq \bar{\bar{z}}_{ih}(t-1) \quad h \in D_i \quad h \notin H_i \\
z_{ih} \geq \bar{\bar{z}}_{ih}(t-1) \quad h \in S_i \quad h \notin H_i
\end{cases}
$$

The result being an optimal price

$p_i^* [p_h(t-1), \bar{\bar{z}}_{ih}(t-1), I_i(t-1)]$

All the description of the dynamic process and equilibrium can be trivially adapted, and we thus omit it.

2. Throughout the paper we assumed that the monopolist used only the preceding period's information in his determination of his perceived demand curve. This was clearly a notation simplifying assumption and we can assume, for example, that the monopolist uses the information he had in the T preceding periods, so that the perceived demand curve will be written

$$
\bar{\bar{Y}}_j [p_j \mid \bar{\bar{y}}_{H_j}(t-1), p_j(t-1), I_j(t-1), \ldots\ldots\ldots\ldots
$$
$$
\ldots\ldots\ldots \bar{\bar{y}}_{H_j}(t-T), p_j(t-T), I_j(t-T)]. \quad (14)
$$

This curve could be obtained by any statistical procedure (Bayesian analysis, regressions...).

The shape of the curve will depend evidently very much on the variables known by the monopolist (The $I_j(t)$). If this information is quite complete, the perceived demand curve may, after a sufficient number of observations, get quite close to the "true demand curve", in particular as for its elasticity.

.../...

(14) Here the consistency condition will be that, if the monopolist has observed the same price and perceived constraint for T periods, the perceived demand curve should go through the corresponding point.

CONCLUSIONS.

Usual analyses of general monopolistic equilibrium were much too reminiscent about the traditional "auctioneer" process, since the monopolist was depicted as taking decisions in function of observations that he actually never had any opportunity to make in the context of the model.

Here in the contrary we have presented a model where transactions can occur outside equilibrium and where observations and decisions can thus be made by decentralized agents (and particularly the monopolistic price setting firms). The price revision mechanism becomes extremely more realistic, and takes into account actually observed variables only.

Also our formulation gives us a much more convincing story to explain the origin of the perceived demand curves family (which is the basis of all monopolistic behavior) : at each date t, the monopolist has a long stream of observations $p_j(\tau)$, $\bar{\bar{y}}_{H_j}(\tau)$, $I_j(\tau)$ for $\tau < t$, from which he can derive the perceived demand curve family by some statistical procedure (regression,..). We see here that the monopolist's information (the set I_j) will be particularly important.

OUr assumption of quantities adjusting infinitely faster than prices was convenient in allowing to separate neatly price and quantity decisions. But clearly the concepts given here would apply as well to the case where prices and quantities adjust together. Though, we should then replace the analysis of "successive" equilibria by an explicit dynamic analysis. This is the subject of a forthcoming paper.

As presented formally, our model deals only with pure monopoly cases. It can be used as well to treat monopolistic competition, if we consider similar products sold by different firms as different economic goods (for location, quality reasons..) but close substitutes (this way of approaching the problem is close to Triffin's one [35] : Cf. his "external interdependence" theory). We also saw that we could treat the problem of product differentiation.

Finally, we must remark that our model, in which each agent takes the actions of the others as given, cannot treat the more general problem of oligopoly, where agents take into account the mutual interdependence of their strategies. For this, more general game-theoretic concepts would be needed (Cf. for example Marschak-Selten [28, Part B]).

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